Financial Contracts and Occupational Choice

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Abstract

Financial constraints and entrepreneurship are among the key factors affecting economic performance in developing countries. Emphasizing the link between the theory microfoundations and the data, the paper considers a heterogeneous agents model of occupational choice with moral hazard under three financial contract regimes differing in their degree of market incompleteness: savings only, borrowing and lending, and insurance. Using maximum likelihood estimation and statistical model comparison methods, I find evidence that the more advanced financial contract regimes allowing for borrowing or insurance provide a better fit with cross-sectional and time-series data from Thailand compared to the savings only regime. However, a direct comparison between the borrowing and lending and insurance regimes shows that neither of them can be rejected in favor of the other relative to the data. Augmenting the contracts with wealth-pooling lottery redistribution arrangements improves further the explanatory power of the model. A new numerical solution technique for incentive-constrained occupational choice models based on non-linear optimization is also proposed.

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1 Introduction

Entrepreneurship is widely recognized as one of the engines of economic growth and development¹. Entrepreneurial activity typically involves investment in capital which can come from two basic sources: own wealth or borrowed funds. Borrowing requires the existence of financial institutions and contracts and is usually subject to moral hazard as the lender cannot observe perfectly the actions of the borrower. The relationship between entrepreneurship and wealth is two-fold: on the one hand wealth inequality affects the pattern of agents' occupational choices and on the other hand, assuming bequests exist, these choices affect next generation's wealth distribution. This implies that the distribution of wealth and the structure and level of development of the financial intermediation system are crucial determinants of the degree of entrepreneurship.

The current paper proposes an occupational choice model of entrepreneurship combining the necessary ingredients listed above: wealth heterogeneity, financial intermediation, and moral hazard. The goal is to identify the relationship among the structure of financial contracts, the wealth distribution, and the pattern of occupational choice in the economy by taking the model predictions to the data. I use numerical simulation, structural maximum likelihood, and calibration techniques to determine what types of financial contracts produce a mapping between wealth and occupational choice which fits best developing country data.

The model in the paper is a substantially generalized version of Aghion and Bolton's (1997) occupational choice model. There is a continuum of risk averse agents who differ in their wealth endowments and entrepreneurial ability and who can choose between two possible occupations: 'entrepreneur', using effort and investment as inputs in production, or 'subsistence worker'², using only effort. Output is stochastic. The effort levels supplied by the entrepreneurs are unobservable to outsiders, which results in a moral hazard problem. The agents can enter into contracts with a competitive financial intermediary (a bank) subject to incentive constraints. Each agent lives for one period in the end of which she bequests a fixed fraction of her wealth to a single child. Due to the indivisibility inherent in the occupational choice, there will be a non-convexity in the agent's indirect utility of wealth function. This suggests that pooling resources among agents and redistributing them through a wealth lottery can be efficiency improving. The mechanism is similar in spirit to the one of rotating savings and credit associations ('roscas'³) observed in developing countries⁴.

Keeping the preference and technology structure of the model unchanged, I characterize its implications under three financial contract regimes: savings only (no credit), borrowing and lending with bankruptcy, and insurance (contingent transfers to and from a financial

¹Knight (1921), Schumpeter (1934), Huff (1993), and Formaini (2001).

²Hereafter simply 'worker'.

³Note, however, that the arrangement described here is technically not exactly the same as a 'rosca' since it is just a one-shot wealth lottery with winners and losers whereas roscas last for many periods and eventually everyone wins the pot.

⁴See Besley, Coate and Loury (1993), Besley and Levenson (1996), Calomiris and Rajaraman (1998) and Klonner (2002) for more detailed theoretical discussion and empirical evidence on roscas in developing countries.

intermediary)⁵. Within each regime I consider the effect of augmenting the contractual structure with wealth lotteries as described above. The resulting six contract types differ significantly in their assumed degree of market incompleteness, ranging from the very restrictive savings only contract without lotteries which precludes any borrowing to the constrained Pareto efficient insurance contract with lotteries.

The first step in characterizing the model implications under the different regimes is solving for the implied constrained optimal contracts, i.e. the optimal consumptions, investments, efforts, and implied occupational choices as functions of wealth. Aiming for more generality and having in mind that the ultimate goal of this paper is taking the model results to the data, I adopt a numerical solution approach. So far the vast majority of the literature on numerical computation of moral hazard models has used linear programming techniques to solve for the optimal contract after transforming the commodity space into a probabilistic space⁶. While having some advantages due to its generality, this method suffers from heavy computational time and memory requirements. In contrast to the existing literature, I design and implement a non-linear computational method based on the 'first order approach'⁷.

An important theoretical result of the paper is demonstrating the equivalence of lotteries on consumption, investment, effort, and output which arise under the insurance regime to an ex-ante lottery over wealth followed by a deterministic contract which takes as given the optimal occupational choice implied by the wealth won in the lottery. This equivalence allows separating the problem of solving for the optimal contract into two stages which, combined with the use of non-linear optimization, leads to a substantial improvement in computational speed⁸ and accuracy.

The results of the computation are used to perform static and dynamic characterization and comparison of the model predictions under the different contractual regimes with special attention to the effect of wealth lotteries. The static analysis demonstrates convincingly that the model regimes have significantly different implications for agents' consumptions, investments, efforts, and probabilities of starting a business given wealth. The differences across the regimes are most pronounced at low wealth levels. Low wealth agents who need to borrow to finance investment and become entrepreneurs benefit most from expanding the contractual structure to include borrowing and/or insurance. Borrowing entrepreneurs with low wealths supply more effort than their counterparts with the same wealth level under the savings and insurance contracts. More generally, *all differences* between the savings only and

⁵The regime evaluation performed in the paper is similar in spirit to the work of Lehnert, Ligon and Townsend (1997) who concentrate mostly on the theoretical implications of different regimes of financial constraints, and also that of Jappelli and Pistaferri (1999) who test the empirical validity of three models of intertemporal consumption choice. However, up to the author's knowledge no one has performed a contract regime evaluation in a class of moral hazard problems.

⁶See for example Phelan and Townsend (1991), Lehnert (1998), Prescott and Townsend (2001a,b), and Doepke and Townsend (2002).

⁷The 'first order approach' for solving incentive constrained problems consists of replacing the incentive compatibility constraint by the first order condition of the agent's maximization problem. Some technical conditions as exhibited in Rogerson (1986) are needed to ensure the validity of the approach to produce a solution to the original principal-agent problem.

⁸Adopting the solution techniques proposed in this paper reduces computational time for the moral hazard problem by a factor of 10 to 20 relative to the linear programming approach. With regards to the maximum likelihood estimation of the model this amounts to waiting just one day for the program to finish instead of two weeks.

the borrowing and lending contracts occur for low wealth levels at which there is borrowing. In addition, the main differences across the regimes in terms of the predicted probability of being entrepreneur as a function of wealth also occur for low levels of wealth: the savings only model predicts no entrepreneurship among poor agents as they cannot provide enough investment, while the other two regimes allow a positive probability of starting a business for such agents.

The main result from the dynamic simulation of the model is that while the higher degree of financial intermediation and insurance is welfare improving in a static sense, the low consumption variability across good and bad states in the insurance contract may lead to lower effort and/or investment, which can have negative dynamic implications for output levels and growth⁹.

The welfare effect of wealth lotteries is always positive as they help some ex-ante poor agents to become entrepreneurs and hence increase overall output, but this can be at the cost of higher degree of wealth inequality over time. This result relates to the numerous calls for poverty 'safety nets' by policy makers and implies that static considerations need to be carefully weighed against dynamic effects working through the wealth distribution.

The fact that the model implications for the relationship between wealth and occupational choice differ significantly under the various contract regimes provides the basis for the empirical strategy used in the paper. I use cross-sectional wealth and occupation data from Robert Townsend's Thai economic survey to identify which of the described financial contracts matches the data best. I perform a structural maximum likelihood estimation of the model parameters in the six contract regimes which is supplemented by a statistical model comparison test establishing whether one can reject one model specification in favor of another. The maximum likelihood function for each regime is constructed by matching the expected probability of becoming entrepreneur generated by the model at every given wealth level in the sample with the corresponding household entrepreneurial status taken from the data. The econometric analysis is performed for the whole sample, as well as for various data stratifications chosen on the basis of region, wealth, financial participation and education.

The model comparison results show that the savings only regime is rejected in favor of both the borrowing and lending and the insurance regimes across most data stratifications, which can be interpreted as evidence for the presence of some degree of borrowing and/or insurance in the data. Nevertheless, the degree of insurance present is not necessarily the maximum possible in the static sense as the restricted borrowing and lending regime cannot be rejected in favor of the more general insurance regime. The importance of including wealth lotteries in the contracts is demonstrated convincingly by the empirical analysis: the model specifications with lotteries provide significantly better fit compared to their counterparts without lotteries.

The empirical evaluation of the contract regimes concludes by performing a dynamic calibration of the model using macroeconomic data from Thailand for the period 1976-1996. The results demonstrate that the insurance regime matches best the time paths of income growth, wealth inequality, and the fraction of entrepreneurs in the data.

The model framework and implications described above relate to several branches of the existing literature. The empirical development literature has argued that financial constraints

⁹Notice that this is a second-best result as the insurance provided in the model is incomplete and it leads to reduced effort due to incentive compatibility reasons.

are among the key factors affecting economic performance in developing countries. Entrepreneurship is one of the main channels through which this effect works. Liquidity constraints arising from credit market imperfections imply that the ability to borrow and hence the probability of becoming entrepreneur depends on agents' own wealth (Evans and Jovanovic, 1989; Holtz-Eakin, Joulfaian and Rosen, 1994a and 1994b; Dunn and Holtz-Eakin, 2000; Paulson and Townsend, 2002). On this basis, it has been argued that expanding the financial intermediation sector facilitates economic growth (King and Levine, 1993; Greenwood and Jovanovic, 1990). Another branch of the empirical literature has focused on the degree of insurance observed in development economies (Townsend, 1994; Udry, 1994; Jakoby and Skoufias, 1998 and Amin, Topa and Rai, 1998). Most formal econometric tests of risk sharing reject the full insurance hypothesis. To identify the reasons for this rejection, it is important to study the underlying financial contract arrangements. On the other hand, various theoretical models of occupational choice have stressed the importance of incomplete information and borrowing constraints in the credit markets in generating a developing process characterized with inequality and, in some cases, 'poverty traps' (Banerjee and Newman, 1993; Piketty, 1997; Aghion and Bolton, 1997; Lloyd-Ellis and Bernhardt, 2000; Ghatak and Jiang, 2002; Gine and Townsend, 2002; and Townsend and Ueda, 2002).

Unfortunately, in many cases the 'communication' between the empirical and theoretical literatures is weak, with empirical work being purely econometric and theoretical work being purely abstract¹⁰. The present paper attempts to address this problem emphasizing the link between the theory underlying the abstract models of economic development and the data. The usefulness of this 'meeting' of the theory with the data is two-fold: on the one hand it identifies what type of theoretical models are empirically relevant, while on the other hand it suggests directions for further improvements in our modeling of economic reality.

Apart from developing a two-stage non-linear method for solving incentive constrained occupational choice models, the paper contributes to the literature in several other aspects. First, I perform a systematic evaluation of distinctively different financial contract regimes which, however, share the same preference and technology structure¹¹. This approach allows an accurate and consistent assessment of the relevance of the contractual structure for the mapping between wealth and occupational choice. Second, the paper introduces an empirical evaluation of the role of wealth lottery arrangements in occupational choice, growth, and inequality. The macro-level analysis of such financial arrangements and their function in facilitating entrepreneurship complements and extends the microeconomic insights from the literature on 'roscas'. Third, the existing literature on incentive-constrained occupational choice models has been restricted mostly to static model evaluation. In contrast, the paper also looks at certain dynamic implications of the three model regimes, employing a mix of calibration and maximum likelihood estimation techniques using Thai national statistics data.

The rest of the paper is organized as follows. Section two describes the model and the financial contract regimes studied. It also contains a description of the numerical computation techniques used to solve the model and a basic characterization of its static and dynamic implications. Section three contains the results of a structural maximum likelihood estimation

¹⁰Albarran and Attanasio (2001) and Paulson and Townsend (2002) who develop and test the empirical implications of structural models of limited commitment are notable exceptions.

¹¹In comparison, Paulson and Townsend (2002) evaluate the empirical relevance of two structurally different occupational choice models from the existing literature.

of each of the model regimes, followed by statistical comparisons between them to establish which regime fits the data best. Section four studies the dynamics generated by the model and compares them to the data using a mix between estimation and calibration of the model parameters. Section five concludes and provides directions for future research.

2 Model Description

The model of the paper is an extended version of the models of Aghion and Bolton (1997) and Lehnert (1998). There is a continuum of agents, $i \in I$ who are heterogeneous in their consumption good endowments (hereafter 'wealths'), a_i , and entrepreneurial abilities, θ_i , and who have preferences given by u(c, z), where c is consumption and z is labor effort. The utility function is assumed strictly increasing in its first and strictly decreasing in its second argument and concave. There are two available technologies in the economy through which the single consumption and investment good is produced. The choice between the technologies is interpreted as choosing between occupations. The first technology involves investing a positive amount, k, and can be written as $p^e(q|\theta, k, z)$, i.e. the probability of achieving output level q given effort z, investment k and the agent's entrepreneurial ability θ . The probability p^e is assumed increasing in all its arguments. I call this technology 'entrepreneurial' and the agents who use it 'entrepreneurs'. The second technology can be operated without making any investment (i.e. k=0) and can be represented as $p^w(q|z)$ - the probability of achieving output q given effort z and no investment. The probability function p^w is increasing in z. The agents using this technology are called 'subsistence workers' or, in brief, just 'workers'. This technology is meant to be interpreted as a subsistence agriculture technology, hence the assumed stochasticity of output. This is a generalization of the Aghion and Bolton (1997) and Lehnert's (1998) models as they assume that zero investment leads to zero output with certainty.

There exists a risk-neutral competitive financial intermediary (bank) with which agents can enter into financial contracts involving borrowing, lending and/or insurance. Except in their input requirements, the two technologies also differ in the information that they provide to the intermediary, i.e. the degree of contractibility with respect to the inputs employed. More specifically, I assume that the effort levels supplied by the entrepreneurs are unobservable by the bank, which results in a classic moral hazard problem in all non-autarkic settings. It is assumed, however, that the intermediary can design contracts, taking ability into consideration and thus disregarding potential adverse selection complications¹². In contrast, workers' efforts are assumed to be observable¹³ and hence can be stipulated directly and enforced through an appropriate contract. Given their wealth, ability, and possible financial contracts with the bank, the agents choose the occupation which would provide them with higher expected utility. Their maximization problem can be solved in two steps: first, compute the expected utility yielded by each occupation and, second, choose the occupation that yields higher expected

¹²Abbring, J., Chiappori, P., J. Heckman and J. Pinquet, (2002) discuss empirical strategies to jointly test and distinguish moral hazard and adverse selection in the data. The current model can be extended to account for adverse selection by adding the relevant constraints to the optimization problems.

¹³This assumption is not very crucial for the results and can be interpreted as stemming from the fact that the subsistence technology involves performing some simple and easy to control tasks.

utility.

For analytical and computational simplicity I assume that there are two possible output realizations for each occupation. For the entrepreneurs, output can take on two values: $q = \theta q_h$ and $q = \theta q_l$, with $q_h > q_l$, whereas for the workers output is either w_h or $w_l < w_h$. I further normalize $q_l = 0$ and $w_l = 0$ which is interpreted as failure of the business project or the agricultural crop. Finally, notice that higher entrepreneurial ability leads to higher output levels, i.e. for the same levels of investment and effort, a more able entrepreneur would achieve higher expected output.

The basic static model described above can be extended to study dynamics in the following way: assume that the agents live for one period and leave a bequest equal to a constant fraction s of their end-of-period wealth to their single child which becomes the latter's wealth endowment¹⁴. Thus there is no population or technological growth and all dynamics are transitionary. This is definitely a shortcoming of the model, however, it is somewhat attenuated by the fact that it is designed for studying developing economies which are presumably not in a steady state.

As stated in the introduction, I am interested mainly in studying the static and dynamic effects of varying the structure of the financial contracts between the intermediary and the agents. In particular, from a static perspective, I study how consumption, effort, investment, and occupational choice depend on the contractual structure. From a dynamic perspective, I look at the effects on inequality, income levels, growth rates, and the fraction of entrepreneurs. The results are used to assess the empirical relevance of the studied contract types.

The model choice was motivated by the findings of Paulson and Townsend (2002) who show that information constrained occupational choice models of the above type tend to fit developing country data better on aggregate compared to the alternative models of Evans and Jovanovic (1989) and Lloyd-Ellis and Bernhardt (2000). Building on this knowledge, the paper concentrates solely on the Aghion and Bolton (1997) model and focuses on investigating what type of financial contracts within its framework match best the occupational choice patterns observed in the data.

2.1 Types of Financial Contracts

The paper concentrates on three alternative types of financial contracts (regimes) which can be signed between the agents and the intermediary, and which differ in their degree of market incompleteness and ability to serve as consumption smoothing devices. The actual contract structure in each of the three cases is imposed exogenously¹⁵.

¹⁴This is a standard, albeit admittedly quite restrictive assumption used in the occupational choice literature (see Banerjee and Newman, 1993 or Piketty, 1997) which can be justified formally by assuming 'warm-glow' preferences as in Andreoni (1989). Endogenizing the bequest/savings decision is therefore a fruitful venue for future research.

¹⁵This is done for simplicity but may be justified by transaction costs, lack of appropriate institutional mechanisms, government intervention, etc. Such a justification, however, remains outside the scope of this paper.

1. Savings Only

The first contract type is a savings (storage) only contract, i.e. no borrowing is possible. Under this contract, agents can only deposit (store) any amount of the consumption good with the intermediary earning a fixed gross return of r in the end of the period. In the static analysis, the interest rate r is taken as exogenously given but this assumption is relaxed later in the paper when I compute model dynamics. Given the assumptions made above, the maximization problem of an entrepreneur with wealth a and ability θ looks as follows:

$$\max_{z,k} p^{e}(z,k)u(c_{h},z) + (1-p^{e}(z,k))u(c_{l},z)$$

$$s.t. \quad c_{h} = \theta q_{h} + r(a-k)$$

$$c_{l} = r(a-k)$$

$$0 < k < a$$

$$(1)$$

where c_h and c_l are the levels of end-of-period wealth¹⁶ in the case of success or failure. The last constraint states that no borrowing is possible, therefore all investment must be selffinanced. Final wealth in each state of the world is simply the realized output plus any return on savings. Despite the unobservability of effort, there is no moral hazard problem under this type of contract as there is no borrowing.

Similarly, a worker solves:

$$\max_{z} p^{w}(z)u(c_{h}, z) + (1 - p^{w}(z))u(c_{l}, z)$$

$$s.t. \quad c_{h} = w_{h} + ra$$

$$c_{l} = ra$$

$$(2)$$

Since there is no investment, all wealth is deposited with the intermediary.

2. Borrowing and Lending With Bankruptcy

This contract represents a standard borrowing and lending arrangement between an agent and the bank: the agent either deposits some amount of money in the bank in the same way as in the savings contract above and earns r, or she can request a loan from the intermediary. In the latter case, the bank announces a repayment amount for each state of the world, which in general depends on the size of the desired loan. The agent takes the repayment schedule as given and decides how much to borrow. I assume that there is limited liability, i.e. consumption must be non-negative. This means that in the case of project failure (zero output), the agent declares bankruptcy (default) and is unable to repay anything back to the bank¹⁷. The bank takes this possibility into account by setting the required repayment under failure to zero and adjusts the repayment due under success by raising the effective interest rate on borrowing in order to break even on the loan. Clearly, the repayment in case of success would depend on the probability of success of the agent's project. Given a required repayment

¹⁶Notice that in the static case end-of-period wealth equals consumption hence the notation used. In the dynamic extension of the model consumption equals (1 - s) times the agent's end-of-period wealth.

¹⁷See Dubey, Geanakoplos and Shubik (2002) for a discussion of the role of default in a general equilibrium setting.

interest rate under success of R, the maximization problem of a borrowing entrepreneur then looks as follows:

$$\max_{k,z} p^{e}(z,k)u(c_{h},z) + (1 - p^{e}(z,k))u(c_{l},z)$$
(3)
s.t. $c_{h} = \theta q_{h} - R(k-a)$
 $c_{l} = 0$

Remember that effort is unobservable to the bank so it must be induced by stipulating the repayment rate R accordingly. I assume that in equilibrium the bank must earn at least its reservation return of r on each loan, i.e. R must solve:

$$R = \frac{r}{p^e(\hat{z}(R), \hat{k}(R))}$$

where $\hat{z}(R)$ and $\hat{k}(R)$ are the solutions of the entrepreneur's problem, (3) taking R as given. The interpretation of the above condition is that to offset the zero return it makes under failure, the bank has to charge an interest rate higher than r in case of success, which happens with probability p^e .

The limited liability assumption introduces an asymmetry between the optimization problems of borrowers and lenders. A lending entrepreneur solves the problem (1) exhibited above without imposing the constraint $k \leq a$. To be consistent with lending, the optimal investment amount k^* at the solution to this problem must not be higher than the agent's wealth a. The workers also solve exactly the same problem (2) as before as they do not invest and thus do not need to borrow from the intermediary. Obviously, the borrowing/lending contract provides more opportunities for consumption smoothing for the agents who choose the entrepreneurial occupation compared to the savings only contract. As such the former contract (weakly) Pareto dominates the latter. However, this contract is also quite restrictive as it provides no insurance in case of project failure.

3. Insurance/Transfers

The third financial contract regime allows all possible types of state contingent transfers between the agent and the intermediary to achieve maximum consumption smoothing. It is well known that, under full information, when the agent is risk-averse and the intermediary is risk-neutral, the optimal contract will be the one providing equal consumption to the agent in both states of the world (success or failure). This is achieved by the agent making a transfer to the intermediary in case of success and the intermediary making a transfer to the agent in case of failure. The contracts between the workers and the bank have this property. However, since entrepreneurial effort is unobservable, full consumption smoothing is not possible for the entrepreneurs as it is not incentive compatible. Indeed, if the agent is promised the same transfers under success and failure, she will always choose the lowest possible level of effort (zero) since effort is costly and output will be always zero. Thus the optimal contract in this case has the bank providing less than full insurance. The insurance contract Pareto dominates the borrowing/lending contract described above because it provides at least partial consumption smoothing.

Because of the assumption of perfect competition in the credit market, the bank's profits must be zero in equilibrium. This allows us to think of the bank as maximizing the expected utility of its customers as a function of their wealth and ability subject to breaking even and incentive compatibility (where applicable). What the bank actually does is set the investment level, k, the transfer levels, c_h and c_l , and recommend an effort level z which maximize the expected utility of the agent. If the bank finds optimal to set k > 0, the agent will be an entrepreneur, while if it sets k = 0, the agent is assigned to be a worker. It is possible that the bank's choice of k take the form of a lottery, i.e. the bank may find optimal to assign k = 0with some positive probability and some positive level of k with the residual probability. In general, the optimal contract might also involve similar lottery assignments of effort or transfer levels for given occupation and state of the world, although it is easy to see that the assumed concavity of the utility function would make such lotteries degenerate at the optimum.

The usual way of writing the insurance/transfers contract described above involves introducing new variables, $\pi(c, q, z, k | a, \theta)^{18}$, corresponding to the probabilities that a particular transfer, $c \ge 0$, output, $q \ge 0$, effort, $z \ge 0$ and investment, $k \ge 0$ allocation is assigned as a function of the agent's wealth, a, and ability, θ . Due to the moral hazard problem, the transfer level in such an assignment will be in general a function of the output level. Notice that these probabilities can be also interpreted as lotteries over different transfer (consumption), output, investment, and effort allocations that are being offered to the agent. Let us call this type of lotteries 'allocation lotteries'. Introducing such lotteries allows us to write the bank's maximization problem for an agent with wealth a and ability θ as the following linear program in the variables $\pi(c, q, z, k)^{19}$:

$$\max_{\pi(c,q,z,k) \ge 0} \sum_{c,q,z,k} \pi(c,q,z,k) u(c,z)$$
(4)

s.t.

$$\sum_{c} \pi(c, \bar{q}, \bar{z}, \bar{k}) = p(\bar{q}|\bar{z}, \bar{k}) \sum_{c, q} \pi(c, q, \bar{z}, \bar{k}) \quad \text{for all } \bar{q}, \bar{z}, \bar{k}$$
(5)

$$\sum_{c,q,z,k} \pi(c,q,z,k)(c-q) = r \sum_{c,q,z,k} \pi(c,q,z,k)(a-k)$$
(6)

$$\sum_{c,q} \pi(c,q,z,k) u(c,z) \ge \sum_{c,q} \pi(c,q,z,k) \frac{p(q|z',k)}{p(q|z,k)} u(c,z') \quad \text{for all } k > 0, \ z, \ z'$$
(7)

$$\sum_{c,q,z,k} \pi(c,q,z,k) = 1 \tag{8}$$

and where

$$p(q|z,k) = \begin{cases} p^e(z,k) & \text{if } k > 0\\ p^w(z) & \text{if } k = 0 \end{cases}$$

$$\tag{9}$$

Let us describe the problem above in a more detailed way. The objective function is simply the expected utility that an agent would get at the assigned allocation (c, q, z, k). The first constraint, (5), ensures that the probabilities constituting the optimal insurance contract, $\pi(c, q, z, k)$, are consistent in Bayes sense with the production technology p(q|z, k). The second

¹⁸See Prescott and Townsend (1984a and 1984b), Phelan and Townsend (1991), Lehnert (1998), Paulson and Townsend (2002), and Karaivanov (2001).

¹⁹See Prescott (1999) for a detailed description and background information on this type of linear programs.

constraint, (6), is the break-even (zero profit) condition, stating that, on average, all outgoing transfers from the bank must equal all incoming transfers. The following constraint, (7), is the incentive compatibility constraint which makes sure that the recommended effort level will be implemented by the agent. Basically, it states that the expected utility of implementing the recommended level of z (the left hand side) must be bigger or equal to the expected utility of deviating to some alternative effort level z'. Finally, the last constraint, (8), ensures that the probabilities sum to one.

2.2 Solution Techniques

This section describes the techniques used to solve the optimization problems for the three financial contracts described above. The problems are solved numerically since closed form solutions cannot be obtained in general. As the purpose of the paper is comparing the implications of the financial contract regimes and taking them to the data, I use more general and flexible parametrizations and functional forms for the fundamentals of the model, instead of aiming for analytic simplicity and risking to obtain only restricted conclusions. The functional forms used to compute solutions are as follows:

$$u(c,z) = \frac{c^{1-\gamma_1}}{1-\gamma_1} - \lambda \frac{z^{\gamma_2}}{\gamma_2}$$

$$p^e(z,k) = \frac{k^{\alpha} z^{1-\alpha}}{1+k^{\alpha} z^{1-\alpha}} \quad \text{and} \quad p^w(z) = \frac{z}{1+z}$$
(10)

The utility function, u(c, z), displays constant relative risk aversion in consumption represented by the parameter, $\gamma_1 \ge 0^{20}$. This is a generalization of the functional forms used in Aghion and Bolton (1997) and Lehnert (1998) who impose risk neutrality ($\gamma_1 = 0$). Allowing for risk aversion has the important consequence of making the agents demand insurance in case of failure, which not all of the financial contracts described above are able to provide. Thus their implications can be distinguished more easily. The other two preference parameters, $\lambda > 0$ and $\gamma_2 \ge 0$, determine respectively the relative disutility of effort and the degree of aversion to variations of effort and also represent a generalization of the quadratic effort cost used in Aghion and Bolton or Lehnert. The production parameter $\alpha \ge 0$ provides further flexibility with respect to the relative importance of investment and effort for achieving the high output level.

The production (probability of success) functions differ from the ones usually used in the computational moral hazard literature by allowing effort and investment to be chosen on the whole positive ray of the real line, $[0, \infty)$ instead of restricting them to lie on a closed interval, e.g. [0, 1]. The advantage of this approach is that one need not worry about corner solutions which can make the interpretation of the results complicated, and which may have no economic meaning endogenous to the model.

Given the above functional forms, the optimization problems for each occupation in the savings only and the borrowing/lending regimes are solved numerically employing non-linear

²⁰As usual, the case $\gamma_1 = 1$ is interpreted as $\ln c$.

techniques based on the quadratic programming approach. The relative simplicity of the problems allows substituting the constraints in the objective and transforming them to relatively standard unconstrained optimization problems and systems of non-linear equations²¹.

2.2.1 Wealth Lotteries

After solving the utility maximization problem for the workers and entrepreneurs, we can derive their indirect utility functions, $v^{E}(a, \theta)$ and $v^{W}(a)$. In this section I assume that ability is fixed and interpret the indirect utility functions as functions of wealth only. Since consumption depends directly on initial wealth, a in a linear way and since u(.) is concave in c, it is clear that, by the envelope theorem, v^E and v^W are locally concave in a for both the savings only and the borrowing/lending regimes²². Given her wealth level, a, an agent would choose the occupation that provides her with higher indirect utility. Thus the utility realization she would actually obtain lies on the outer envelope of $v^{E}(a)$ and $v^{W}(a)$, i.e. $v(a) \equiv \max\{v^{E}(a), v^{W}(a)\}$. Even though v^E and v^W are concave in wealth, v(a) will not be concave in general²³. The reason for the non-concavity is the indivisibility inherent in the occupational choice problem an agent is assumed to be able to hold only one occupation at a time, i.e. she cannot split her time endowment between the two occupations. Clearly then, an ex-ante lottery in wealth can restore concavity²⁴. Basically, what such a lottery does for an agent with wealth a that puts her in the non-concave region of v(a), is offer her, with some probability, a wealth level, $a_2 > a$. at which she would be on the right concave region of v and, with the residual probability, a wealth of $a_1 < a$, at which she would be at the left concave region of v (see fig. A.1).

Obviously, the lottery would provide the agent with higher expected utility. A natural counterpart for such wealth lotteries exists in various developing countries in the form of the so-called 'rotating savings and credit associations', or 'roscas', which allow individuals to pool their wealths and then assign the pooled wealth by a lottery to one of them to buy a durable good or implement an investment project. Roscas have been studied extensively in the development literature both theoretically and empirically 25 . Technically, the wealth-pooling arrangement in the model is not a typical 'rosca' as the latter usually lasts more than one period. Nevertheless, I think that the parallel is useful and intuitive. In the model, we can think of the wealth lotteries as having several individuals with a given wealth level, a, at which v(a) is convex, pool their wealths and then some fraction of them is assigned via lottery to a higher wealth level and the rest to a lower wealth level, such that the total pooled wealth is exhausted. Notice that the wealth lotteries create inequality, i.e. if we take a number of agents with equal wealths and they play the lottery, as a result they are split into two groups ('losers' and 'winners') with potentially very different wealth levels. This shows that such type of welfare improving arrangements can lead to increased inequality, i.e. equity and efficiency do not align perfectly in this case.

 $^{^{21}}$ More details on the numerical computation methods used, including the actual Matlab codes used to compute the solutions, are available from the author upon request.

 $^{^{22}}$ In the borrowing/lending regime a non-concavity may occur at the wealth level at which agents are indifferent between being borrowers and lenders (see fig. 2b).

²³Unless one of the occupations provides higher utility for all wealth levels.

²⁴Lehnert (1998) and Rosen (2002) also discuss this property of discrete choice models.

²⁵See Besley, Coate and Loury (1993), Besley and Levenson (1996), and Calomiris and Rajaraman (1998).

In the remainder of the paper, I look at the implications of the model regimes described above when ex-ante wealth lotteries are or are not allowed. The no-lottery contracts simply state that the agent chooses the occupation that provides her with higher utility given her wealth but she does not have the opportunity to make this wealth higher by pooling with other agents and entering a lottery. Thus, I am interested to see whether allowing for the agents to participate in wealth pooling arrangements can help alleviate the constraints imposed by the structure of the feasible financial contracts with the intermediary. The welfare effect of the lotteries and their effect on inequality are studied as well.

2.2.2 Equivalence of Allocation Lotteries and Wealth Lotteries

Let me now describe in more detail the technique used to solve the insurance/transfers problem since it represents a new approach for solving moral hazard problems with lotteries. The commonly used in the existing literature method of solving these types of problems numerically is to choose discrete grids for the possible values that c, z and k can take and solve the resulting constrained linear program with respect to the probabilities $\pi(c, q, z, k)$. Although this method is very general and does not rely on almost any assumptions about the functional forms used, it has several major drawbacks. First, even with very modest grid sizes, e.g. 10 points each, the dimension of the problem expressed in the number of variables and constraints is very high²⁶. This 'dimensionality curse' requires a lot of computer memory and time, especially when one wishes to use denser grids. On the other hand, if too coarse grids are used, the quality of the solution deteriorates and the results may be unreliable. A second shortcoming of the linear programming approach is that discretizing the problem using grids may introduce so-called 'grid-lotteries' which arise when the true solution is between two of the grid points. These lotteries have no economic interpretation and may 'contaminate' the solution. Finally, if the grid end points turn out to be chosen incorrectly, it is possible to obtain a corner solution when in fact the true solution is different. Because of these reasons, I use the particular structure of the occupational choice model and propose a new method of solving such moral hazard problems with lotteries.

The main innovation in my approach is that it transforms the original problem into an equivalent one which involves solving two non-linear constrained optimization problems, one for each occupation. The basic idea is to use a result similar to the one of Cole and Prescott (1993) which shows that solving the linear program with lotteries, (4), is equivalent to the two-stage process of solving the optimization problems for the two occupations separately and then computing the necessary lottery over wealth only, as described above.

Under the insurance regime, the problem of the bank contracting with an entrepreneur can be written as follows:

$$\max_{k,z,c_h,c_l} u^*(z,k) \equiv p^e(z,k)u(c_h,z) + (1-p^e(z,k))u(c_l,z)$$
(11)

s.t.
$$z \in \arg\max_{z} u^*(z,k)$$
 (ICC)

$$p^{e}(z,k)c_{h} + (1 - p^{e}(z,k))c_{l} = r(a-k) + p^{e}(z,k)\theta q_{h}$$
 (BE1)

²⁶With 10 grid points each for c, z and k and 2 output levels, the number of variables in the program is 2000 and there are 1102 constraints.

The interpretation is that the bank sets k, c_h and c_l and recommends an effort level z, which maximize the expected utility of the entrepreneur subject to two constraints: first, the recommended effort level must be indeed the optimal one chosen by the agent given k, c_h and c_l , and second, the bank must break even, i.e. the expected outlays (the left hand side of (BE1)) must be equal to the expected income (the right hand side). Intuitively, what happens is that the agent commits to giving all her wealth and output to the bank²⁷ and in exchange obtains transfers of c_h or c_l depending on the state of the world.

The problem of the intermediary contracting with a worker is similar, except that there is no need of the incentive compatibility constraint (ICC) as effort is fully observable. Due to the concavity of the utility function, it is optimal to set consumption and effort equal in the two states. Thus we can write the problem of the bank simply as^{28} :

$$\max_{z,c} u(c,z) \tag{12}$$

s.t.
$$c = p^w(z)w_h + ra$$
 (BE2)

The second constraint is simply (BE1) written for this problem since now $c_h = c_l$ and k = 0.

The following result provides the basis of the method used to solve for the optimal contract in the insurance regime.

Proposition 1 (Equivalence Between Allocation Lotteries and Wealth Lotteries)

Assume that the agent's utility function is separable in effort and consumption, strictly concave in consumption and strictly convex in effort. Then the optimal effort, investment, and consumption levels²⁹ corresponding to the solution of the linear moral hazard program (4) (the 'allocation lottery problem') coincide with the solutions of the optimization problems (11) and (12) combined with an ex-ante lottery over wealth only (the 'wealth lottery problem').

Proof:

The idea for the proof is based on Proposition 5 in Cole and Prescott (1993). Basically, we need to show that for any given wealth level, a, the contract (c, q, z, k) resulting from the solution of the allocation lottery program can be mapped into the solutions of the optimization problems (11) and (12) combined with an ex-ante lottery over wealth. The unique (because of concavity) solution to the allocation lottery program is the set of probabilities³⁰ $\{\pi^*(c, q, z, k | a) > 0^{31}\}$ satisfying the constraints in (4), whereas the solution to the wealth lottery problem can be written as $(c_j^*(a_1), z^*(a_1), k^*(a_1)), j = l, h$ and $(c_j^*(a_2), z^*(a_2), k^*(a_2)), j = l, h$ together with a probability $\mu^*(a)$ such that

$$a_1(1 - \mu^*(a)) + a_2\mu^*(a) = a \tag{13}$$

²⁷Output and wealth are observable and it is assumed that it is not possible to hide any of them.

²⁸Note that I could have written the problem in its general form as in (11) and then show that the solution satisfies $c_h = c_l$. I take a shortcut instead and impose the latter condition to make the problem simpler.

²⁹I.e. the optimal contracts between the agents and the intermediary.

 $^{^{30}\}mathrm{I}$ will refer to this set as the set of optimal contracts.

³¹I disregard zero probability contracts as they have no economic interpretation.

and where c_j^* , k^* and z^* are the solutions to (11) and (12). It is also clear that q in $\pi^*(c, q, z, k|a)$ can take only the values $0, \theta q_h$ and w_h due to technological feasibility.

Notice that, given our assumptions about the preferences (separability plus strict concavity) and the production function, the problems (11) and (12) have unique solutions in terms of c, z, k for any given value of a. Also, by the envelope theorem, the indirect utility functions $v^{E}(a)$ and $v^{W}(a)$ are concave, thus when wealth lotteries are used for convexification the losers and the winners of the lottery would have different occupations. Suppose that, without loss of generality, an agent with wealth a_2 would optimally choose to be an entrepreneur (i.e. $v^{E}(a_2) > v^{W}(a_2)$), whereas an agent with wealth a_1 would optimally choose subsistence work. Let us denote $\Pi_1 \equiv \{\pi^*(c, q, z, k | a) \mid k = 0\}$, to be the set of contracts under which the agent is a worker and $\Pi_2 \equiv \{\pi^*(c, q, z, k | a) \mid k > 0\}$, to be the set of contracts under which the agent would be an entrepreneur³².

Suppose that there exist two optimal contracts $\pi_1^*(c_1, q_1, z_1, k_1)$ and $\pi_2^*(c_2, q_2, z_2, k_2)$ in Π_2 , such that their corresponding effort/investment assignments are not the same, i.e. $(z_1, k_1) \neq (z_2, k_2)$. As u and p are concave in z and k, this would imply that a linear combination of the two would achieve higher utility for the entrepreneur and still be feasible which is a contradiction. Thus it must be the case that $z_1 = z_2 = z^1$ and $k_1 = k_2 = k^1$ implying that there are only two elements in Π_2 , $\pi_{21}^*(c_h^1, \theta q_h, z^1, k^1)$ and $\pi_{22}^*(c_l^1, 0, z^1, k^1)$. Similarly, there are only two elements in $\Pi_1 : \pi_{11}^*(c_h^2, w_h, z^2, 0)$ and $\pi_{12}^*(c_l^2, 0, z^2, 0)$.

Now we only have to show that $(c_j^i, z^i, k^i) = (c_j^*(a_i), z^*(a_i), k^*(a_i))$ for j = l, h, i = 1, 2to finish the proof. Define $\hat{\pi}_{11} = (1 - \mu^*(a))p(w_h|z^*(a_1), 0), \hat{\pi}_{12} = (1 - \mu^*(a))p(0|z^*(a_1), 0), \hat{\pi}_{21} = \mu^*(a)p(\theta q_h|z^*(a_2), k^*(a_2)), \text{ and } \hat{\pi}_{22} = \mu^*(a)p(0|z^*(a_2), k^*(a_2)).$ It can be seen immediately from (BE1), (BE2) and (ICC) together with (13) that the vector $(\hat{\pi}_{11}, \hat{\pi}_{12}, \hat{\pi}_{21}, \hat{\pi}_{22})$ satisfies (6) and (7). It also satisfies (5) and (8) by construction thus it is feasible for the linear program (4). Conversely, (5) implies that $\frac{\pi_{11}^*}{p(w_h|z^2, k^2)} = \frac{\pi_{12}^*}{p(0|z^2, k^2)} \equiv \mu_1$ and $\frac{\pi_{21}^*}{p(\theta q_h|z^1, k^1)} = \frac{\pi_{22}^*}{p(0|z^1, k^1)} \equiv \mu_2$. Then from (8) we have that $\mu_1 + \mu_2 = 1$, i.e. it is clear that (z_i^*, k^i) , the implied c_j^i , and μ_1 and $1 - \mu_1$ satisfy the constraints of (11) and (12) together with (13) and thus are feasible for the wealth lottery problem. Finally, suppose that $(c_j^*(a_i), z^*(a_i), k^*(a_i))$ together with μ_1 and $1 - \mu_1$ is not maximizing for the wealth lottery problem, i.e. some agent can achieve higher utility. But then the mapping of $(c_j^*(a_i), z^*(a_i), k^*(a_i))$ and μ^* into the π -contracts described above would produce an allocation in which all agents are at least as well off as in $(\pi_{11}^*, \pi_{12}^*, \pi_{21}^*, \pi_{22}^*)$ which is a contradiction.

The proposition states that under some mildly restrictive assumptions we can replicate the solution to the allocation lottery problem (4) by using only a wealth lottery among the agents with a given wealth level, a. This implies that the only contribution of the allocation lotteries is to allow agents to engage in implicit lotteries over wealth. The agents find it optimal to participate in such wealth lotteries as their indirect utility of wealth has convex regions induced by the indivisibility of occupational choice.

Showing the equivalence between the solutions of the allocation lottery and the wealth

 $^{^{32}}$ It can never be optimal to assign k = 0 for an entrepreneur as this implies zero output at any effort level, whereas if the agent were assigned to be a worker at the same effort level, she would produce positive output. Thus all agents assigned entrepreneurship will have k > 0.

lottery problems as described above is the first step of the computational algorithm used to solve for the optimal insurance/transfers contract. The main purpose of this step is to reduce the dimensionality of the problem by making possible to solve the problems of agents of different occupations separately. Moreover, it was demonstrated that under our assumptions wealth lotteries are sufficient to reproduce the optimal contracts, i.e. we can disregard any other types of possible allocation lotteries.

The next question is how to solve the problems (11) and (12) and how to compute numerically the wealth lottery which is equivalent to convexifying the v(a) function. One possibility is to use the linear programming approach separately for each occupation but this has all the disadvantages discussed above. Instead, I solve the problems in their non-linear form. The worker's problem (12) is a standard non-linear maximization program and can be solved by conventional optimization methods. To solve the entrepreneur's problem (11), however, we need to transform the incentive compatibility constraint (ICC) into a more manageable form. The standard way to do this is to replace it by the first order condition of the maximization problem with respect to effort. This is known as the 'first order approach'. As demonstrated by the literature³³, this approach is by no means universally valid and requires some restrictive properties to be satisfied by the probability function defining output as a function of effort. The next result shows that the first order approach is valid under our assumptions, i.e. the solution obtained by replacing the maximization problem in (ICC) with its first order condition is a maximum of the objective function of (11).

Proposition 2 (Validity of the First Order Approach)

The production function $p^e(q|z,k)$ satisfies the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) implying that the first order approach is valid.

Proof:

Let us first verify the MLRP which means that we need to prove that $\frac{\partial p^e(q|z,k)}{\partial z} \frac{1}{p^e(q|z,k)}$ is non-decreasing in q. Since there are only two possible levels that q can take in the model, we simply need to show that:

$$\frac{\partial(1-\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}})}{\partial z}\frac{1}{1-\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}} \leq \frac{\partial(\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}})}{\partial z}\frac{1}{\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}}$$

which is obviously true as the left hand side is negative and the right hand side is positive.

Now let us verify the CDFC which is equivalent to showing that $\frac{\partial^2 p^e(q_1|z,k)}{\partial z^2}$ and

 $\frac{\partial^2 p^e(q_1|z,k)}{\partial z^2} + \frac{\partial^2 p^e(q_2|z,k)}{\partial z^2}$ are non-negative where q_1 and q_2 are the two possible output levels and $q_1 < q_2$. The first expression is equivalent to:

$$\frac{\partial(-\frac{(1-\alpha)k^{\alpha}z^{-\alpha}}{(1+k^{\alpha}z^{1-\alpha})^2})}{\partial z} = \frac{\alpha(1-\alpha)k^{\alpha}z^{-\alpha-1}(1+k^{\alpha}z^{1-\alpha})+2(1-\alpha)^2k^{2\alpha}z^{-2\alpha}}{(1+k^{\alpha}z^{1-\alpha})^3} > 0$$

 33 See for example Mirlees (1975) and Rogerson (1985).

We also have:

$$\frac{\partial^2 p^e(q_2|z,k)}{\partial z^2} = \frac{\partial(\frac{(1-\alpha)k^{\alpha}z^{-\alpha}}{(1+k^{\alpha}z^{1-\alpha})^2})}{\partial z} = -\frac{\partial^2 p^e(q_1|z,k)}{\partial z^2}$$

thus the second expression is non-negative as well. Given these results, Proposition 1 in Rogerson (1985) implies that the first order approach is valid in our setting.

The proof of the sufficiency of the MLRP and CDFC for the validity of the first order approach is due to Rogerson (1985). In general, in a setting with more than two output levels, the two conditions become more restrictive but they can still be satisfied by carefully specifying the production function. Notice that the numerical solution method proposed in the paper does not necessarily require the use of the first order approach since a significant improvement in computational speed and memory usage is achieved already by decomposing the problem into two smaller problems, one for each occupation. Using the first order approach, however, allows us to speed up the computation even further and to abstract from grid issues.

Having shown that the first order approach is valid, we can replace the (ICC) by the first order condition $\frac{\partial u^*}{\partial z} = 0$ which is a non-linear equality constraint in z, c and k and use non-linear optimization methods to solve the problem. The only remaining issue is the wealth lottery, which is equivalent to convexifying $v(a) = \max\{v^E(a), v^W(a)\}$ by taking its upper convex hull, $v^{C}(a)$. This step is performed by choosing a dense discrete grid on wealth a, computing the value of v(a) at the grid points by solving directly the non-linear problems (11) or (12), and then computing the upper convex hull³⁴ of the points with coordinates $(a_i, v(a_i))$ where $a_i, j = 1, ...n$ are the points in the grid. After the convex hull is computed, we can evaluate it at any wealth level a by using a cubic spline approximation³⁵. Knowing the convex hull of v, however, gives us only the indirect utility value obtained by an agent with a given wealth a. To get the actual optimal contract between the agent and the bank, we also need to know at which values for a (if any) wealth lotteries will be used at the optimum. This amounts to solving for the points A and B on fig. A.1. In the numerical algorithm these two points are found by comparing the values of the convex hull function $v^{C}(a)$ with the maximum function v(a) and taking the first and last grid point at which they differ. Once A and B are computed, we know that all agents with $a \in (a_1, a_2)$ optimally participate in a wealth lottery with payoffs the wealth levels a_1 and a_2 . The probabilities of getting each level are backed out from the distances among a, a_1 and a_2 . The rest of the agents do not participate in wealth lotteries as the indirect utility function is concave at their wealth endowments.

The proposed method of solving for the optimal insurance/transfer contract described above has the following advantages over the standard linear programming method used in the existing literature. First, no grids are used in the optimization, which reduces the memory and computational time requirements. The relative performance in terms of computational speed is about ten to twenty times higher for the non-linear approach proposed here compared to the linear programming approach using standard non-commercial maximization routines

 $^{^{34}}$ The convex hull computation method is based on the Quickhull algorithm (see Barber, Dobkin and Huhdanpaa, 1996).

³⁵The full details of the numerical algorithm plus the program codes that implement it are available from the author upon request.

and average grid sizes. Second, using non-linear methods improves the solution precision as the optimization is done on continuous as opposed to discrete sets of values. Third, lotteries are used only and exactly when they are needed, in contrast to the allocation lottery formulation in which grid lotteries are prevalent and can affect the results. Finally, the results of the optimization do not come out in the form of the artificial objects $\pi(c, q, z, k)$ which are harder to interpret from an economic point of view. Instead, we directly obtain the assigned consumption transfer, investment and effort levels. A potential disadvantage of the proposed method is that it relies on more restrictive assumptions compared to the linear programming method which could limit its applicability in some particular settings.

As in the savings only and the borrowing/lending regimes, it is also possible to solve for the optimal insurance contract when wealth lotteries are not present (i.e. when the agents or the intermediary cannot implement rosca-type arrangements). In the allocation lottery interpretation this is equivalent to restricting the possible allocation lotteries to the values of one and zero only³⁶. Thus, in total, there are six possible financial contract regimes which are studied in the paper³⁷:

- 1. Savings only contract without wealth lotteries (SNL)
- 2. Savings only contract with wealth lotteries (SL)
- 3. Borrowing and lending contract without wealth lotteries (BNL)
- 4. Borrowing and lending contract with wealth lotteries (BL)
- 5. Insurance contract without wealth lotteries (INL)
- 6. Insurance contract with wealth lotteries (IL)

2.3 Model Implications

This section describes the results of a numerical simulation of the model using the functional forms described above. All six types of financial contracts are considered. I start with some static properties of the model solution and then turn to the model's dynamic implications. The main purpose of the section is to demonstrate the salient features of the model and to provide a clear picture of the differences among the contract regimes with regards to consumption, investment, effort, and occupational choice from a static point of view and output, inequality, and the level of entrepreneurship from a dynamic point of view. Table B.1 lists the parameters at which the model is simulated for each contract type. The parameter values correspond to the maximum likelihood estimates of the parameters of the insurance with lotteries (IL) contract which is found to fit the data best. A detailed description of how these estimates are obtained is available in the next section. The fact that the model is simulated using the estimated parameter values allows us to be more confident of the empirical relevance of the results.

2.3.1 Statics

Let us first look at the static implications of the model. Figures A.2, A.3 and A.4 characterize the model solution for the three contract types. In the top panels of fig. A.2, we see the

 $^{^{36}}$ For more details see the discussion in Lehnert (1998) who follows this approach using the linear programming solution method.

³⁷The abbreviations in the brackets refer to the regime names used in the empirical section and the tables.

indirect utility functions for workers and entrepreneurs in each of the three regimes. They are all globally concave with the exception of the one for the borrowing entrepreneurs which consists of two concave parts. The non-convexity occurs at the point where agents shift from being borrowers to lenders. The graphs illustrate clearly the case for Pareto improving wealth lotteries as described above. Notice also the potential role of lotteries in convexifying the non-concave portion of the borrowing/lending (B/L) indirect utility function.

Consumption is depicted in the second and third rows of graphs in fig. A.2. We see that, under the insurance contract, the gap between consumption in the high and low states is the smallest. Under the B/L contract consumption in the zero output state is zero for low wealth levels. The insurance contract provides positive consumption under failure for the low wealth entrepreneurs and full consumption smoothing for the workers which is efficiency improving. Notice the discrete jump in consumption under success in the B/L contract when agents shift from borrowing to self-financing and the effective interest rate faced by them drops from R to r.

Fig. A.3 shows the behavior of effort, investment, borrowing, and the probability of being entrepreneur as functions of agent's wealth. Entrepreneurial effort and investment are both increasing in wealth due to their complementarity in production and the fact that investment becomes relatively cheaper as more wealth is accumulated. Notice that low wealth entrepreneurs exert more effort and invest more relative to the levels they would have supplied under self-financing. The intuition is that these agents are borrowers and they have bigger incentive to exercise more effort as they become able to invest more since they want to minimize the probability of bankruptcy and getting zero consumption. The limited liability clause in the borrowing and lending contract and the higher repayment rate charged by the bank as a result induce a higher effort level from such agents. Worker's effort decreases in wealth in all model regimes implying a strong wealth effect - the richer the workers become, the less they have to work and the higher their demand for leisure. Once again, notice the discontinuous drop in effort and investment when agents shift to self-financing in the B/L contract and no longer face the incentive compatibility constraint.

Looking across the contract types, observe that entrepreneurial effort is on average highest under the borrowing and lending regime and lowest in the insurance one. This is due to the fact that, given the consumption smoothing provided by the intermediary, the agents do not have to work as hard to avoid the bad state of the world. By construction, no borrowing is possible in the savings only setup, whereas agents with low wealths are being lent money by the intermediary under the insurance and the B/L contracts and can potentially become entrepreneurs at lower wealth levels. Borrowing is decreasing in wealth as agents turn to self-financing trying to decrease the impact of the incentive compatibility constraint.

The six model regimes have significantly different predictions about the expected probability of being entrepreneur³⁸ as a function of wealth (see the bottom panels of fig. A.3). The main differences occur for low wealth levels. We see that the SNL regime predicts zero entrepreneurs for all wealth levels up to .2, whereas the corresponding cutoff is only .05 for the BNL regime and all agents may become entrepreneurs with positive probability in the insurance setting. Augmenting the financial contracts with lotteries helps the low wealth agents

³⁸The probabilities were computed by integrating over the talent distribution instead of holding θ fixed in order to be more consistent with the empirical part of the paper.

by giving them the opportunity to become entrepreneurs if they win the lottery. The effect of adding lotteries is strongest in the insurance regime. Notice also that increasing the level of financial intermediation by moving across the three contracts increases the probability of starting a business. The probability is increasing in wealth in general due to the existence of financial constraints in the model.

Figure A.4 depicts the differences in the indirect utilities achieved under the three financial contract regimes, including the cases when wealth lotteries are and are not allowed. We see that, once again, the biggest differences occur for the low wealth financially constrained agents, especially in the savings and borrowing/lending settings. As expected, allowing for wealth lotteries improves the welfare of only the agents with wealths in the non-concave portion of the upper envelope of the indirect utility functions for the two occupations. The insurance contract provides strictly higher indirect utility for both occupations (especially at low wealth levels) due consumption smoothing. The borrowing/lending contract also dominates the savings only contract for the entrepreneurs, with this effect being strongest at low wealth levels as agents are allowed to borrow.

To measure in real terms the magnitude of the differences in utility obtained under the various contractual structures at the given parameters, I compute the utility equalizing consumption supplements which would make an agent indifferent between two contractual regimes (see Table B.2). For example, we see that banning wealth lotteries in the insurance setup would require a 0.06% raise in consumption on average (over all wealth levels) and 0.85% for the agent who is most affected, to make her indifferent to the case when lotteries are allowed. The same numbers are two to three times lower for the other regimes. We see that the direct welfare improving effect of lotteries is not very strong at the chosen parameter values but alternative simulations show that this effect can be much stronger, especially if risk aversion is higher. The comparison across the three types of financial contracts reveals larger utility differentials. On average, to move an agent from a borrowing and lending to a savings only world would require an average consumption supplement of 0.1% (0.58% maximum), whereas the same shift starting from the insurance contract would require 2.8% additional consumption on average, but more than 13% for the worst affected agents³⁹. These numbers imply that more complex financial contracts have the potential to lead to significant welfare gains at least in a static sense.

2.3.2 Dynamics

Let us now move to the dynamic implications of the model⁴⁰. To characterize the dynamics, I have simulated the model economy under the six different contractual regimes listed in the previous section. The parameters used are again the ones from Table B.1. The only difference is that the interest rate, r, is now taken as endogenous in the simulations, i.e. it is chosen to make borrowing equal to lending in each of the regimes in order to get more realistic dynamics. The savings rate, s, is set to .25 which is in the ballpark of the levels observed in Thailand. All agents (500 in the simulation) start with zero wealth.

³⁹The compensating differentials can be significantly higher (up to 70%) for other parameter configurations.

⁴⁰Since the model lacks realistic dynamic elements like endogenous savings rate or capital accumulation, the resulting dynamics are quite simplistic and are provided mainly as an illustration.

Looking at figures A.5-A.7 we observe rising income, investment, consumption, and number of entrepreneurs over time which matches qualitatively the patterns observed in growing developing countries. Investment is lowest in the savings only regime as all of it is self-financed. Allowing for wealth lotteries makes the most dramatic difference to the results in the savings only setup - without lotteries, output converges to a lower level, investment is significantly reduced and fewer agents become entrepreneurs. This implies that, apart from their static welfare effects, wealth lotteries can be also welfare improving in a dynamic sense. This is demonstrated in the average utility panel on fig. A.5 as well. This improvement, however, comes at the cost of a higher wealth inequality level. Comparing the Gini coefficients in the lottery and no-lottery economies, we see that inequality is about 10% higher in the former. The intuition for this result is that lotteries create inequality by shifting agents with ex-ante equal wealths to different wealth levels.

Under the borrowing/lending and insurance contracts (fig. A.6-A.7), we observe an increase in borrowing over time, as well as an increase in the ratio of borrowing to output (not depicted) especially in the early periods. This implies an increase in the degree of financial intermediation in the model economy which matches stylized development facts. Wealth inequality is also rising as the economy grows, matching the Thai experience. The wealth lotteries do not seem to have a dramatic effect on the results for these two contract types, perhaps because few agents fall into the relevant wealth range. Nevertheless, the lottery economies do perform slightly better than the no-lottery ones, especially in the insurance case.

Comparing across the contractual regimes we see that the highest output and investment levels are achieved under the borrowing and lending contract and the lowest are observed under the savings only regime. The difference in the maximum levels of output attained is around 20% with lotteries and 30% without lotteries. In contrast, the average utility level is the highest in the insurance contract, followed by the B/L and savings only ones. These results come to warn us that the highest output levels are not always equivalent to the highest welfare levels. Thus economic policies aimed solely at raising output may actually lead to lower utility on average. The reason for this misalignment of what we usually believe to be equivalent policy objectives is that the average effort and investment levels under the insurance regime are lower compared to the B/L regime as agents' incentives to work hard are mitigated by the consumption smoothing provided by the intermediary. This leads to a lower probability of success and, hence, lower achieved output levels. I conclude that enriching the financial contracts structure is welfare improving in the dynamic sense⁴¹ as well, although it may not necessarily lead to maximum output.

Entrepreneurship is also strongly affected by the form of the financial contract available in the economy. The borrowing and lending contract gives rise to the highest levels of entrepreneurship (30%), closely followed by the insurance contract (25%), while the SNL regime features much less businesses (10%). With regards to inequality, both the savings only and the B/L regimes have much higher Gini coefficients (0.66 to .76) compared to the one observed under the insurance contract which peaks at 0.25. The reason for this result is the consumption smoothing inherent in the insurance regime which brings the levels of consumption and savings (and hence next period's initial wealths) of successful and unsuccessful agents closer

⁴¹Note, however, that one must take these results with some caution as the model in the paper is not a full-blown dynamic model but rather a dynastic model with exogenously fixed saving rate.

to each other. The different degree of consumption smoothing across the contracts can be seen also in the consumption panels of figures A.5-A.7.

To summarize, I have shown that the three contract types have significantly different implications with respect to various economic variables including the probability of entrepreneurship as a function of wealth. The differences are most pronounced at low wealth levels. These results provide the basis for the next section in which I perform a maximum likelihood estimation and statistical comparison tests of the model regimes aiming to identify which of them fits best the wealth-occupation pattern observed in the Thai data. The fact that the regime predictions differ mostly for low wealth agents is helpful as the wealth distribution in the data is heavily skewed to the right.

3 Structural Maximum Likelihood Estimation

To broaden our understanding of how differences in the structure of the available financial contracts affect agents' occupational choices, I make an empirical evaluation of the goodness of fit of the six model regimes presented above using data on village households in Thailand. First, I perform a structural maximum likelihood estimation of each regime and obtain estimates for the parameters used. The estimation is done for the whole sample and also for various sub-samples of households. The parameter estimates are then analyzed in terms of the implied relationship between the model and the data. Next, the six contractual structures are compared pairwise in order to identify the regime that comes closest to the data generating process.

Here is the place to acknowledge the potential limitations of the structural estimation method: in some cases it may be hard to distinguish testing of the imposed model structure versus testing the actual relationships between the data variables we are interested in. The paper tries to address these concerns by using flexible functional forms and by imposing a consistent preference and technology structure across the model regimes. Nevertheless less structural semi- or non-parametric methods of looking at the data, as well as various other robustness checks, should definitely be of high priority on the future research agenda.

3.1 Data and Estimation Methodology

To perform a maximum likelihood estimation of the model, I use data from Robert M. Townsend's socioeconomic survey of Thai villages⁴². The survey was fielded in 1997 in four provinces located in different regions of Thailand. The sample used in this paper consists of 2313 households, about 14% of which run a business. Consistent with the model, a business is defined as basically anything different from subsistence farming. The households in the sample originate from two distinct parts of the country: the rural and semi-urban central region close to the capital and the much poorer and more traditionally rural northeastern region. The survey data includes information on household wealth, occupational history, access to and use of various formal and informal financial institutions, and detailed demographic and educational characteristics. Economic theory suggests that characteristics of this type are

⁴²For a more detailed description of the data see Paulson and Townsend (2002), Binford, Lee and Townsend (2001), and also Robert Townsend's website: http://www.src.uchicago.edu/users/robt

important determinants of the household's decisions regarding supply and demand of credit and choice of occupation. The paper uses the variability in household characteristics to test the relative performance of the model regimes, anticipating that different model setups will perform best in the different data stratifications.

The sample focuses only on businesses started within a five-year period prior to the survey, which was done to obtain a more accurate assessment of the process of transition into entrepreneurship. Consistent with that, the household wealths in the data correspond to six years prior to the survey date, i.e. to wealths prior to choosing whether to become an entrepreneur. All non-positive wealth observations, as well as the outlier observations from the top wealth percentile were removed from the original dataset.

Table B.3 presents a statistical summary of the data, with separate sections for the central and North-East regions. We see that the distribution of household wealth is highly skewed to the right with the median much lower than the mean. Thus the sample is characterized with relatively few very rich households and many relatively poor ones. This would help us distinguish better among the model regimes since the biggest differences in their implications occur exactly at low levels of wealth. The fraction of entrepreneurs is 14% for the whole sample and it crucially depends on education, with only 9% of the low education agents running a business, while this number among the ones with more than 4 years of education is 21%. The mean wealth of agents with financial access is about two times higher than that of agents without financial access which can be treated as evidence for borrowing being dependent on wealth and/or for the existence of borrowing constraints for poorer households. The agents with financial access are also characterized with a higher fraction of entrepreneurs (16%) compared to the ones without financial access (12%). Finally, entrepreneurs are on average two times richer than workers. As already mentioned, the North-East region is much poorer than the central one (the mean wealth is 3.5 times lower) and features very high skewness of the wealth distribution. There are also much less entrepreneurs in the North-East compared to the central region (9% versus 19%). As in the aggregate data, in both regions entrepreneurship is strictly increasing in the level of education and access to credit.

Figure A.8 which is taken from Paulson and Townsend (2002) displays the results of a non-parametric regression of the probability of starting a business on wealth. We see that the relationship between wealth and the probability of becoming entrepreneur in the data is positive in general which is a symptom of financial constraints. We already saw that the model in the paper is able to provide such a relationship. Two effects working in opposite directions can be responsible for the particular shape of this relationship. On the one hand, an increase in wealth allows higher investment and makes entrepreneurship more attractive since a higher probability of success can be achieved. On the other hand, however, higher wealth means that the agent can consume out of her interest income and does not have to put in high effort so she may choose to be a rentier "worker" instead (wealth effect). Depending on which of the two effects dominates, entrepreneurship can potentially decrease or increase with wealth.

Apart from wealth, the other important determinant of the probability of starting a business is the agents' entrepreneurial ability. The model accounts for ability through the parameter θ which has been held constant so far, but which is allowed to vary in the estimation procedure below. More specifically, I assume that talent is unobserved by the econometrician but follows a known distribution characterized by parameters which are to be estimated. Formally, the talent variable, θ , is assumed to be continuously distributed on the unit interval $[\kappa, \kappa + 1]$ with a probability density function $\eta(\theta) = 2m(\theta - \kappa) + 1 - m$, where $\kappa \ge 0$ and $m \in [-1, 1]$ are parameters which will be endogenously determined during the estimation. The parameter m characterizes the shape of the talent distribution. When m is equal to zero, talent is uniformly distributed. When m is one, more mass is put on high ability agents, whereas for m equal to -1, most of the mass is put on low-talent agents. Notice that despite the model assumption that talent is independent of wealth, we can still study the interaction between these two variables by looking at different data stratifications, taking wealth as given. The independence assumption is without doubt restrictive but notice that even if wealth is endogenous (e.g. if we assume that high ability agents have accumulated more wealth), financial constraints would still matter and the results would still differ across the contract types.

I test the empirical relevance of the model by performing a maximum likelihood estimation of the probability of being entrepreneur generated by the model and the observed household occupational status from the data. The log-likelihood function can be written as:

$$L(\phi) \equiv \frac{1}{N} \sum_{i=1}^{N} E_i \ln H(a_i | \phi) + (1 - E_i) \ln(1 - H(a_i | \phi))$$

where N is the number of observations, E_i is a binary variable which takes the value of 1 if agent i is an entrepreneur in the data and 0 otherwise, a_i is the wealth of agent i in the data, ϕ is a vector of model parameters as described below, and $H(a_i|\phi)$ is the expected probability of being entrepreneur generated by the model for an agent with wealth a_i integrated over the unobserved talent variable.

The probability of being entrepreneur generated by the model, $H(a_i|\phi)$, is a function of the following parameters which are the elements of ϕ^{43} :

 γ_1 - the coefficient of risk aversion;

 γ_2 - a curvature parameter of the disutility of effort;

 λ - a multiplicative constant governing the relative weight of utility derived from consumption or leisure;

q - a parameter determining the higher value of output for the entrepreneurs;

- α the investment share in the probability of success function;
- m a parameter governing the shape of the talent distribution;
- κ a parameter determining the support of the talent distribution;
- w a parameter determining the higher value of output for the workers;
- r the interest rate;

The first eight parameters are estimated, meaning that the log-likelihood function is maximized with respect to them. The interest rate is set to 1.25 as found in the Thai data. The likelihood maximization is performed separately for each of the six model regimes described above as the implicit function $H(a_i|\phi)$ differs across them. Given a wealth level $a = a_i$, the model generates a value for the probability that a person with such wealth is an entrepreneur. Maximum likelihood estimation is then used to recover the model parameters which provide the best match between these generated probabilities and the actual occupation status observed in the data. To ensure identification, the wealth levels from the Thai data were normalized to lie on the interval (0, 1].

 $^{^{43}}$ See also the description of the model above.

The numerical procedure used to solve the likelihood maximization problem comprises of the following steps⁴⁴. First, for any parameter vector, ϕ , and any given values for θ and a, we need to solve the relevant non-linear optimization program and compute the implied probability of being entrepreneur, $h(a|\phi,\theta)$. In this step, I use extensively the results of Propositions 1 and 2 to optimize the numerical solution procedure using the proposed twostage non-linear computation method. Second, since θ is assumed to be unobserved by the econometrician, we need to compute the expected value of $h(a|\phi,\theta)$ integrating over θ , i.e. $h^e(a|\phi) \equiv \int_{\kappa}^{1+\kappa} h(a|\phi,\theta)\eta(\theta)d\theta$. The method used is Gauss-Legendre quadrature with 5 nodes for θ (see Judd, 1998). It represents a commonly used numerical integration technique performed by evaluating the integrand at suitably selected nodes in the support. The method was chosen because it minimizes the number of function evaluations and also because of its nice asymptotic properties.

Due to computation time considerations, I cannot afford to compute $h^e(a|\phi)$ at all data points for wealth (2313 in the whole sample), so I construct a 20-point non-uniformly spaced grid⁴⁵ on [0,1] and compute $h^e(a|\phi)$ only at the grid points. In order to be able to compute the probability for all wealth points in the data, I use a cubic spline interpolation on the grid, which generates the probability of being entrepreneur predicted by the model for given wealth level a, $H(a|\phi)$.

The actual maximization of the log-likelihood function, $L(\phi)$, is done as follows. First, in order to ensure that a global maximum is reached, I perform an extensive grid search over the eight parameters and pick the parameter configuration which maximizes L as the vector of initial parameter values for the actual optimization procedure. Second, given this initial guess, I solve the non-linear optimization problem of maximizing L to obtain the maximum likelihood estimate ϕ^* . The solution procedure represents a generalization of the polytope method using the Nelder-Mead simplex algorithm. It was chosen because of its high reliability, relative insensitivity to different initial values, and good performance with low-curvature objective functions, such as those that usually appear in multivariate likelihood maximization problems.

The standard errors for the parameter estimates are computed by taking an approximation of the parameter covariance matrix, using the outer product of gradients (OPG) method, M = SS'/n, where S denotes the $n \times 8$ matrix of score vectors, $S_{.j} = \frac{\partial L(\phi)}{\partial \phi_j}$, j = 1..8with respect to the estimated parameters. The standard errors of the estimated parameters are then the square roots of the main diagonal elements of the matrix M^{-1}/n . The score vectors themselves are computed using one-sided numerical differentiation of $L(\phi)$ around the maximizing parameter values with tolerance of 10^{-3} , i.e. the actual derivative $\frac{\partial L(\phi)}{\partial \phi_j}$ is approximated by $\frac{L(\phi') - L(\phi)}{h}$, where $h = 10^{-3}$ and $\phi' = (\phi_1, .., \phi_j + h, .., \phi_8)$.

⁴⁴Paulson and Townsend, 2002 (see the appendix) use the same numerical estimation algorithm which was designed and implemented by the author.

⁴⁵The unequal spacing between the grid points is needed since the wealth data is heavily right-skewed.

3.2 Estimation Results

The results of the structural maximum likelihood estimation of the six model regimes are presented in tables B.4 - B.9. I have chosen to report results for the eight most important data stratifications: by region, wealth, and education, although numerous others were also estimated as a robustness check.

Looking at the parameter estimates, we see that, in the cases when standard errors are low, risk aversion γ_1 tends to be lower for high-wealth stratifications in most model regimes. Risk aversion also tends to decrease with education although there are some exceptions. In most of the cases the parameter is fairly accurately estimated, implying that its variations across the data stratifications are statistically significant. With the exception of the SL case, the estimated risk aversion for the whole sample is relatively low - around .1. It is much higher, however, for some of the low wealth and North-East stratifications, as well as for the SL regime.

In most of the model regimes the effort disutility curvature parameter γ_2 is increasing in the level of education indicating higher aversion to changes in the effort level. In the lottery regimes, γ_2 is also increasing in wealth. Interestingly, the estimated value for γ_2 in the whole sample increases as we move towards financial contracts providing more insurance and credit: it is around .25 in the savings only contracts, .6 in the borrowing and lending ones, and 1.5 in the insurance setup. This is another illustration of the structural differences across the three basic regimes: different parameters values are needed to produce occupational patterns similar to the ones in the common data. Allowing for wealth lotteries does not seem to affect significantly the curvature parameter estimates.

The effort disutility parameter λ tends to increase on average with education although the direct effect of wealth on it is not very significant in most of the cases. The results demonstrate that there is variation in the preference parameters of the model across the different data stratifications, which is used to uncover the inherent relationship between wealth, ability and financial constraints in the data.

The production function parameter α is, in most cases, estimated to be slightly higher for high wealth agents, showing that they use investment to a greater extent since it is relatively cheaper for them compared to effort. Notice that the estimated investment share in production, α , is also generally increasing in education. In most cases the ratio between the high output levels for the entrepreneurs and the subsistence workers, q/w is estimated to be around or above 2, which is consistent with the data for the mean wealths of entrepreneurs and non-entrepreneurs exhibited in table B.3.

Finally, let us look at the two parameters in the model, m and κ , which govern the support and shape of the talent distribution. For the whole sample in the B/L and the savings only with lotteries regimes, m is close to zero, implying a uniform talent distribution. In contrast, in the remaining cases, m is strongly negative, implying that more mass is being put on low talent agents. However, many of the standard errors for m are quite large so these conclusions are not always statistically significant. Looking at the results for κ , we see some confirmation of the intuitive trend for higher parameter estimates as education increases, although the non-monotonic relationship between wealth and entrepreneurship in the data may force the optimal values of k to go in a different direction. In most cases the parameter is also increasing in wealth. Looking at the likelihood values, we see that all models achieve best fit in the North-East region and among the low education households, while worst fit is achieved in the central and the high education stratifications. This is mainly due to statistical reasons as the model performs better in stratifications with fewer entrepreneurs due to the lower degree of variation in the data.

3.3 Model Comparisons

In this section, I build upon the maximum likelihood results from above by performing a formal statistical test of how well the six model specifications fit the occupational choice pattern in several stratifications of the data. The six regimes are compared against each other in pairs using the Vuong likelihood ratio test for non-nested models (see Vuong, 1989). The main advantage of the method is that it does not require that any of the models which are being compared be the true model that has generated the data. The null hypothesis of the test is that the two compared models are equally likely to have generated the observed data. The Vuong method involves computing a likelihood ratio test statistic which, under certain conditions that we assume to hold and after a suitable normalization, is distributed as a standard normal random variable.

The results of the model comparisons are exhibited in table B.10. I consider 14 data stratifications ⁴⁶. I have added a few more household groups to the eight discussed above, dealing mostly with access to different types of formal and informal financing and debt. The purpose is to try to distinguish better the implications of the different model regimes about how occupational choice varies with wealth, education, and access to different forms of credit. The table contains the results of ten bilateral comparisons across the six model specifications. First, I compare all no-lottery and all lottery models between each other, which is followed by a test of the effect of including wealth lotteries within each of the three basic financial contract regimes. We see that the savings only with no lotteries setup (SNL) is rejected to be equally likely to have generated the data when compared to both the borrowing and insurance regimes in the whole sample and four other data stratifications. This provides evidence that the extreme form of financial restrictions which it imposes is far from the data generating process. The savings only regime also provides worse fit with the data when compared to borrowing and insurance in virtually all of the remaining data categories⁴⁷. This is an evidence that households in the data do have some ability to borrow or obtain insurance transfers from financial intermediaries, unlike the assumption of the savings only setup. A similar result is true for the saving only contract with lotteries although the comparison statistic is significant for fewer data categories. The rejection of the savings only regime by the data is intuitive: remember that it predicts that, since no borrowing is possible, investment is limited and all low wealth agents must be workers. In contrast, the other two regimes allow borrowing, hence entrepreneurship might be preferred even at low wealths.

In general, the test cannot reject the null hypothesis that the borrowing/lending and the insurance contracts are equally likely to have generated the wealth-occupation pattern ob-

⁴⁶Fifty data stratifications in total were estimated to check the robustness of the reported results.

 $^{^{47}}$ The only exception is the group of agents with education >4 years, where the comparison statistic is insignificantly positive. A possible explanation is that these agents are also among the richest and thus are able to self-finance investment and save.

served in the data in virtually all stratifications. This implies that my results differ somewhat from those of Paulson and Townsend (2002) who find that limited liability constraints as in the borrowing model are more important (i.e. provide better fit with the data) at low wealth levels, while constraints due to moral hazard as in the insurance contract are more important at higher wealth levels. Of course, no direct parallel between the results in the two papers can be drawn as Paulson and Townsend compare two models with completely different structure in terms of production functions, preferences, etc. (the Evans and Jovanovic, 1989 and the Aghion and Bolton, 1997 models) while the model regimes evaluated in the present paper are much closer to each other and all feature asymmetric information, unlike the Evans and Jovanovic model.

The fact that we cannot reject any of the borrowing and lending and insurance regimes in favor of the other can be interpreted as evidence for incomplete insurance in the village economies, which is consistent with the findings of Townsend (1994). The results suggest that the degree of insurance in the economy observed through its effect on occupational choice is higher than what is implied by the savings only contract but lower than the maximum possible level which would prevail under the insurance/transfers contract. Another important implication of the model comparison results is that the borrowing and lending and the insurance contracts seem to have similar ability to provide agents with credit and consumption insurance at least measured by the likelihood of the predicted occupational choice pattern⁴⁸.

Notice, however, that the savings only contract without lotteries is not rejected in the low wealth stratifications with fewer entrepreneurs (Wealth below median, North-East, Education < 4 years and North-East, access to BAAC⁴⁹). This can be interpreted as indirect evidence for the existence of more restrictive financial constraints operating for these groups of households. It is also an evidence that a lower degree of development as in the North-East region is more likely to be characterized with a lower degree of insurance and more restricted financial contracts.

Three of the model comparisons in table B.10 study the effect of adding lotteries to each of the financial contracts. From the theoretical results above, we know that allowing wealth lotteries has the potential to change substantially the model implications about occupational choice at different wealth levels. In particular, agents with wealth levels at which the upper envelope of their indirect utility functions for the two occupations is convex can be assigned to each occupation with some positive probability in a lottery regime, whereas if no lotteries are present they would be assigned to just one of the occupations with probability one. The table shows that there is considerable evidence that contracts with lotteries provide better fit with the data compared to their counterparts without lotteries. The result is most evident for the savings only contract. Although quite a few of the Vuong likelihood ratio statistics for the borrowing and insurance setups are not significant at the 10% level, the vast majority of the signs are negative, indicating that lottery contracts achieve better fit with the data. An exception is the group of households from the North-East with access to BAAC financing for which the contract without wealth lotteries performs better, albeit non-significantly. The reason for this may be the specific lending practices employed by the BAAC in this region

⁴⁸Dubey, Geanakoplos and Shubik (2002) show that allowing for default in financial contracts like the B/L regime can increase the dimension of the asset span thus moving the allocation closer to the constrained Pareto optimum corresponding to the insurance regime in the current paper.

⁴⁹The Bank for Agriculture and Agricultural Cooperatives (BAAC) is a dominant lender in Thai villages.

as discussed in Paulson and Townsend (2002). We see a similar result in the SNL vs. INL comparison as well: while the restrictive SNL contract is rejected in the central region, it cannot be rejected in the North-East, showing once again that stronger financial constraints are likely to be more relevant for the poorer households of the North-East. The SNL regime is also strongly rejected for agents holding debt but not for the ones with no debt, which is consistent with the fact that no borrowing is possible in the savings only setup. A similar result holds in the SL vs. BL comparison.

Here is the place to emphasize that one should not take the above results as direct evidence that there should necessarily exist rosca-type wealth lottery arrangements which operate in the Thai villages. Remember that I have shown in the theoretical section of the paper that such lotteries can be equivalent to allocation lotteries over consumption, investment, effort and output, i.e. what the estimation procedure picks may be the latter financial arrangements instead of wealth lotteries.

The last comparison in table B.10 looks at borrowing and lending with lotteries (BL) versus insurance without lotteries (INL). The idea is to test whether the strength of the effect of adding lotteries on the probability of starting a business differs between the two types of contracts. The results show that the INL regime is rejected in favor of the BL one in the rich central region and for agents with wealth above the median. In contrast, the insurance regime provides better fit (albeit insignificantly) in the low wealth North-East, Wealth below median and North-East, BAAC stratifications. These findings suggest that more formal contracts, such as borrowing and lending, have more explanatory power in accounting for the occupational choice patterns for richer households, whereas informal arrangements featuring insurance are more important for poorer households.

An alternative way to compare the model regimes' ability to fit the occupational choice pattern observed in the data is to generate the predicted number of entrepreneurs at different wealth levels for each regime evaluated at the respective estimated parameter values and compare these predictions with the actual numbers from the data. This method gives us more descriptive information than that contained in the likelihood value by indicating why and for what wealth levels the alternative model regimes succeed or fail in matching the data⁵⁰. Table B.11 exhibits the actual and predicted number of entrepreneurs for each of the six regimes computed by wealth quartiles. For example, the numbers in the data column imply that there were 80 actual entrepreneurs among the poorest quarter of the agents, 67 among the next quarter, etc. We observe that the SNL and SL regimes fail to match the data for different reasons - the former matches well the first quartile but then greatly overpredicts the number of entrepreneurs, while the latter severely underpredicts the degree of entrepreneurship in the last quartile. Confirming the likelihood comparisons, the borrowing and lending and insurance regimes do a much better job in matching the data although none of them seems to be able to replicate the decline in the actual number of entrepreneurs in the second and third quartile at the estimated parameters.

⁵⁰However, notice that the method is used only for illustrative purposes since it does not provide us with a way to statistically compare across the model regimes and, in addition, the objective of the likelihood maximization is different and more complex than simply matching the four numbers in the data column of Table B.11. For example notice that the INL regime matches the total actual number of entrepreneurs perfectly but nevertheless achieves only the third highest likelihood value.

4 Dynamics and Calibration

This section performs an empirical analysis of the dynamic implications of the model under the different financial contract regimes. The goal is to evaluate the fit between the time series of output growth, wealth inequality and entrepreneurship, generated by the model and their counterparts from the Thai data. This would supplements the results from the crosssection obtained above and show to what extent the model can explain dynamic patterns in the data in addition to static ones. Because of the overly simplistic and mechanical way the model dynamics are generated, the results in this section should be taken with some caution but nevertheless should provide a useful starting point for a more complete and realistic dynamic analysis of entrepreneurship featuring optimal endogenous savings and/or capital accumulation decisions.

The maximum likelihood estimation performed in the previous section provides estimates only for the static parameters of the model - the preference parameters, γ_1, γ_2 and λ , the production parameters α, q and w, and the talent distribution parameters m and κ . However, to study the capability of the different model regimes to match the dynamics of the Thai economy, we also need to pin down certain dynamic parameters. More specifically, in this section I assume that q grows at a net rate of g each period thus the two dynamic parameters of the model are g and the savings rate s. The method used to determine these two parameters is calibration, i.e. I search for the best (g, s) combination which, combined with the maximum likelihood estimates of the static parameters, produces dynamics that come closest to the data, according to a suitably chosen metric⁵¹. The calibration process matches the following three time series generated by each of the model regimes with Thai macroeconomic data:

- the growth rate of output,
- the Gini coefficient of wealth inequality,
- the fraction of entrepreneurs in the economy.

Due to computational reasons, I restrict attention to time series of six periods generated by each of the model regimes⁵² which are matched with Thai National Statistics data from the period 1976-1996. Thus, one period in the model is assumed to be equivalent to four years in the data. The data for the income growth rate, Gini coefficient, and the fraction of entrepreneurs in Thailand were taken from successive rounds of the national income and expenditures Socio-Economic Survey (SES) conducted by the Thai government. A more detailed description of this dataset can be found in Gine and Townsend (2002), Jeong (1999) and Jeong and Townsend (2002).

To perform the calibration, I use as a metric the normalized sum of period by period squared deviations of the three time series generated by the model from the actual Thai data. The normalization is done by dividing the deviations by their means from the data. Formally, I use the following metric, proposed by Gine and Townsend (2002):

$$\frac{1}{3} \sum_{j=1}^{3} \sum_{t=1976}^{1996} \left(\frac{z_{st}^{sim} - z_{st}^{data}}{\mu_{z_s^{data}}}\right)^2$$

 $^{^{51}}$ The approach is similar to the one used in Gine and Townsend (2002).

 $^{^{52}}$ To keep the setup as general as possible, I concentrate only on the regimes with wealth lotteries.

where z_{st} denotes the variable s (one of the three time-series being matched) at time t and $\mu_{z^{data}}$ is the mean of the variable z_s^{data} .

In order to be able to simulate the model, I also need data for the initial wealth distribution. This data comes from the 1976 round of the SES. I use only young households data⁵³ as suggested by the structure of the model. The sample size is 1,325 observations. Some descriptive statistics of the data are exhibited in table B.12. We see that they are qualitatively similar to those of the Townsend dataset. The wealth figures are lower in nominal terms since the data is for 1976 rather than 1997 as in the Townsend survey.

Finally, to be consistent, I need to re-estimate using maximum likelihood the eight 'static' parameters listed above from the 1976 SES data⁵⁴. These parameters are needed to generate the simulated time series for income growth, inequality and the fraction of entrepreneurs. The results of the estimation are reported in table B.13. A significant difference from the previous results is the good fit achieved by the savings only model specification which cannot be rejected by the Vuong test at a 10% confidence level. Moreover, the savings only contract achieves a slightly, albeit insignificantly better, likelihood value compared to the two other regimes (see table B.14). Of course, we should not forget that this data refers to 1976 when the financial intermediation sector in Thailand was much less developed compared to 1997 so perhaps this result should not come as a surprise⁵⁵.

The parameter values from table B.13, together with the initial wealth distribution in 1976, are used to simulate the three lottery model specifications, SL, BL and IL for five four-year periods which correspond to 1980-1996. The results of the calibration can be found in table B.15 and figure A.9. In terms of the chosen metric, the best fit is achieved by the insurance specification, followed by the borrowing/lending one. Both of them match relatively well the initial pattern of the income growth rate, although the growth rates achieved in the later periods of the simulations are lower than the ones in the data⁵⁶. The BL model matches quite well the degree of wealth inequality but predicts no entrepreneurs. In contrast, the IL model matches well the fraction of entrepreneurs but underestimates the degree of wealth inequality over time. For most of the time periods, the SL model predicts zero entrepreneurs at the estimated and calibrated parameters which is below the actual fraction in the data and it also predicts a slightly lower inequality than observed. The parameter estimates for the savings rate range between .29 and .46 (see table B.15), which is relatively close to the actual savings rate of 22-33% in Thailand for the period. The parameter g has no direct counterpart in the data and does not seem to play an important role in the calibration at least for the given values of the 'static' parameters.

⁵³I.e. data on households who might have become entrepreneurs only recently.

⁵⁴As a robustness check, I have also simulated the model dynamics, using the parameter estimates obtained in the previous section from the 1997 Townsend data. As expected this produced significantly worse fit.

⁵⁵Indeed, when the estimation is done using 1996 SES data the BL regime provides the best fit, which is consistent with the financial deepening process observed in Thailand.

⁵⁶This is an inevitable deficiency of the model as decreasing returns set in at high output and investment levels.

5 Conclusions

The recent negative economic experiences of some Asian and Latin American developing countries who have seemed to follow the 'right' macroeconomic policies have called for a reconsideration of the way policy recommendations are made. The focus has shifted towards studying the microeconomic underpinnings of such economic crises and identifying the key ingredients of the economic development process operating at individual or household level. Particularly important issues are the effects of these events on wealth inequality versus growth and on the welfare of the poorest part of the population.

Financial constraints, wealth inequality, and differences in entrepreneurial talent are some of the most important factors influencing economic development. This paper uses a model of occupational choice under moral hazard featuring agents, heterogeneous in wealth and ability to analyze the implications of differences in the degree of financial intermediation available in the economy on consumption, investment, effort, income growth, inequality, and entrepreneurship. Three financial contract regimes were studied and compared: a savings only regime, a borrowing/lending regime, and an insurance/transfers regime. The regimes differ in their assumed degree of financial markets incompleteness, and thus in their ability to serve as consumption smoothing devices.

Agents in the model can choose between two occupations, which amounts to operating one of two different technologies. The indivisibility inherent in the occupational choice generates a non-convexity in the indirect utility function, which makes pooling wealth among agents and then redistributing it via lottery efficiency improving. The mechanism underlying these wealth lotteries is similar to the one of the rotating savings and credit associations (roscas) operating in many developing economies. To account for the existence of such informal institutions, the paper considers the effects of including wealth lotteries in each of the three regimes listed above.

I have analyzed the implications of differences in the structure of financial contracts on the model economy from both a theoretical and an empirical point of view, using advanced numerical solution techniques. The model simulations show that while the insurance regime achieves the highest utility in any given time period, it is also characterized by lower levels of effort, which can potentially have negative dynamic effects on output. I have also demonstrated that it is possible to achieve huge gains in income and/or welfare by switching to more advanced financial contracts. This suggests that institutional improvements in the financial intermediation sector of developing countries can have dramatic effects on the well-being of their citizens. Allowing for wealth lotteries also leads to welfare improvements in both static and dynamic sense, however, this may be at the cost of increased wealth inequality.

The main theoretical result of the paper is demonstrating the equivalence of lotteries on consumption, investment, effort and output in the insurance regime to an ex-ante lottery over wealth followed by a non-probabilistic insurance contract *given* the optimal occupational choice implied by the wealth won in the lottery. This equivalence allows a significant simplification of the numerical computation method for incentive-constrained occupational choice problems over the commonly used in the literature linear programming approach. In addition, this method can be easily applied to any similar type of discrete choice problems.

Entrepreneurship, viewed as an engine of economic growth, is one of the main points of interest in the present paper. The model simulations show that entrepreneurship is strongly

affected by the form of financial contracts available in the economy. More restrictive financial regimes limit considerably the fraction of agents that become entrepreneurs due to borrowing constraints. The potential of the model to generate differences in its implications with respect to occupational choice under the three financial contracts is used extensively in the empirical part of the paper where I perform a maximum likelihood estimation of the various model regimes matching the generated occupational choice patterns with those observed in the Thai data. One of the main empirical results is the rejection of the savings only contract in favor of the more general borrowing and lending and insurance contracts. This is an evidence for the existence of consumption smoothing contractual arrangements in the data although not necessarily at the incentive constrained Pareto optimal level. This is true since the Vuong likelihood ratio test cannot reject the null hypothesis that the more limited borrowing/lending regime is equally likely to have generated the data as the more general insurance/transfers regime.

Testing formally for the empirical relevance of including versus not including wealth lotteries in the financial contracts represents a contribution to the existing literature. The paper demonstrates that augmenting each of the three contract regimes with wealth lotteries improves the fit with the data. The effect is especially strong for the savings only contract, providing some indirect evidence for the existence and importance of asset pooling arrangements in developing countries, such as Thailand.

The results described in the paper give rise to some important policy implications. We saw that increasing the degree of financial intermediation in the economy is unambiguously welfare improving with the biggest gains achieved by low wealth households. Expanding the existing financial contracts available to the poor to include credit and insurance would help some of them start businesses, which can raise aggregate output and consumption and improve allocative efficiency. The theoretical and empirical findings also demonstrate that wealth lottery arrangements similar to roscas can contribute in increasing welfare and efficiency as well. Providing institutional and financial support to such informal organizations in developing countries should then prove to be beneficial.

Some caution must be exercised with regards to the above policy implications of the model. Technically they would be valid only if the assumptions I have made about the source of information incompleteness are correct, i.e. if the main problem preventing the attainment of the first best allocation is moral hazard. For instance, if limited commitment were the predominant information imperfection in Thai villages, it may be unwise to extend credit to poor individuals given that they may be likely to renege on the loan contract.

Several potential directions exist in which the results presented above can be improved and extended. First, I have considered only three of the numerous possible types of financial contracts. Although they were carefully chosen because of their similarity to many financial arrangements observed in developing countries, other types of contracts do exist and deserve attention. Second, the type of financial contract present in the model economy at any given time is exogenously set. Certainly, a more general framework allowing for endogenous changes in the level of financial intermediation over time as in Townsend and Ueda (2002) would move the analysis closer to reality. Allowing the agents in the model to sign multiple contracts of different types at the same time would further improve the model. Third, in the empirical part of the paper I have used only wealth and occupational status data to perform the structural maximum likelihood estimation of the model. Incorporating data on business income and wages can undoubtedly enrich the current results. Fourth, in the present model I assumed for simplicity's sake that entrepreneurial talent is unobservable to the econometrician and thus I let the estimation procedure determine if any correlation between wealth and talent exists. An alternative methodology would be to impose some structure on this relationship and use the implied wealth-talent feedback in the estimation. Finally, semi- and non-parametric estimation methods can be used as a robustness check of the results.

To summarize, in this paper I analyzed various static and dynamic implications of different financial contracts for the process of economic development and evaluated which contractual structures are best suited to explain the patterns observed in the data. This 'marriage' between theory and data should be able to enhance our ability to make better empirically backed policy recommendations based on the microeconomic underpinnings of the economic processes involved.

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Figure A.1: Wealth Lotteries



Figure A.2: Static Model Implications I



Figure A.3: Static Model Implications II



Figure A.4: Utility Differences



Figure A.5: Dynamic Model Implications, Savings Only



Figure A.6: Dynamic Model Implications, Borrowing and Lending



Figure A.7: Dynamic Model Implications, Insurance



Source: Paulson and Townsend (2002)

Figure A.8: Non-Parametric Relationship Between Starting a Business and Wealth



Figure A.9: Model Calibration, SES Data (Best Overall Fit)

Parameter	Value
γ_1	0.1012
γ_2	1.5167
λ	0.3934
α	0.8088
q	1.0017
θ	2.3000
W	0.5034
S	0.2500
r*	1.2500

* The interest rate r is endogenously determined in the dynamic simulations.

Table B.1: Simulation Parameters

	Average	Maximum
No Lottery vs. Lotttery		
Insurance	0.06%	0.85%
Borrowing/Lending	0.03%	0.24%
Savings Only	0.03%	0.23%
Contract Comparison		
Savings Only vs. B/L	0.10%	0.58%
Savings Only vs. Insurance	2.80%	13.56%
B/L vs. Insurance	2.73%	13.21%

Required Increase in Consumption

Table B.2: Utility Equalizing Consumption Supplements

	M	hole Sample			Northeast			Central	
	business	no business	total	business	no business	total	business 1	no business	total
Observations	320	1993	2313	111	1111	1222	209	882	1091
Percent of sample	14%	86%		9%6	91%		19%	81%	
Mean wealth (1000 baht)	1466	653	765	451	380	386	2004	266	1190
St. dev. wealth (1000 baht)	2815	1450	1727	514	681	668	3342	1989	2342
Median wealth (1000 baht)	280	200	205	215	180	180	306	245	253
Wealth skewness	2.99	5.29	4.82	1.38	7.41	7.19	2.27	3.8	3.41
Min wealth (1000 baht)	1	0.02	0.02	1	0.02	0.02	1.2	0.5	0.52
Max wealth (1000 baht)	16400	15000	16400	2134	12600	12600	16400	15000	16400
Mean wealth (with fin. access)	1770	917	1055	535	473	479	2362	1413	1618
Mean wealth (no fin. access)	1107	436	515	366	311	315	1549	609	767
Percentages									
Education < 4 years	9%6	91%	20%*	5%	95%	20%*	13%	87%	19%
Education $= 4$ years	14%	86%	e7%*	9%6	91%	68%*	20%	80%	66% *
Education > 4 years	21%	79%	13%*	16%	84%	12%*	25%	75%	15%*
Wealth below median	13%	87%		8.8%	91.2%		17%	83%	
Wealth above median	15%	85%		9.3%	90.7%		21%	79%	
Financial access	16%	84%	46%*	11%	89%	43%*	22%	88%	50%*
No financial access	12%	88%	54%*	8%	92%	57%*	17%	83%	50%*
* Percentage of total number of observa	ations in the 1	egion.							

Table B.3: Summary Statistics, 1997 Townsend Data

Stratification	logL				Parameter Est	imates			
		γ_1	γ_2	۲	α	б	ш	X	W
Whole sample	-0.4121	0.0914	0.2443	0.1258	0.1836	0.8801	-0.7156	0.8706	0.8894
		(0.0002)	(0.0002)	(00001)	(0.0003)	(00000)	(0.0425)	(00000)	(00000)
Central	-0.4884	0.0981	0.2712	0.0962	0.1555	0.9443	-0.6990	0.7820	1.0161
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0506)	(0.0003)	(00000)
Northeast	-0.3193	1.3243	0.3010	0.0939	0.1996	1.0164	-0.0002	0.2073	0.2018
		(00000)	(00000)	(0.0591)	(00000)	(00000)	(0.0966)	(0.0591)	(00000)
Wealth below median	-0.3792	1.2414	0.2593	0.0930	0.1953	1.0688	0.0013	0.1829	0.2374
		(0.0001)	(00000)	(00000)	(00001)	(00000)	(0.2890)	(00000)	(00000)
Wealth above median	-0.4112	0.1586	1.5929	0.7435	0.1717	1.0338	-0.0097	0.2682	0.5771
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0781)	(0.0001)	(00000)
Education < 4 yrs	-0.3008	1.3765	0.3194	0.0939	0.1820	1.1175	-0.3152	0.2200	0.2257
		(00000)	(0.5424)	(00000)	(00000)	(0.3127)	(0.1208)	(00000)	(00000)
Education = 4 yrs	-0.4144	0.0888	0.2441	0.1260	0.1835	0.8785	-0.7255	0.8786	0.8875
		(00000)	(00000)	(0.0028)	(0.0005)	(0.0001)	(0.0454)	(0.0001)	(00000)
Education > 4 yrs	-0.5066	0.0990	0.2887	0.0995	0.1855	1.0321	-0.5573	0.7898	1.0456
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1026)	(0.0006)	(00000)

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Table B.4: Savings Only, No Wealth Lotteries

Stratification	logL				Parameter Est	imates			
		γ_1	γ_2	У	α	d	ш	У	M
Whole sample	-0.3966	1.2907	0.3142	0.1008	0.1933	1.1596	0.0016	1.0998	0.1944
I		(0.0011)	(0.0061)	(0.0023)	(0.0014)	(00000)	(0.0013)	(00000)	(00000)
Central	-0.4808	1.2525	0.3111	0.0996	0.2014	0.9584	0.5609	1.4963	0.2008
		(3.9621)	(0.0017)	(2.5414)	(3.0622)	(0.0017)	(0.0034)	(0.0058)	(00000)
Northeast	-0.3083	0.0665	0.8062	0.5262	0.2084	2.5566	0.0049	1.4529	0.1327
		(00000)	(00000)	(00000)	(00000)	(00001)	(0.0084)	(00000)	(00000)
Wealth below median	-0.3793	0.0470	1.0706	0.1923	0.1942	0.4001	0.0090	0.0134	0.0827
		(00000)	(00001)	(00000)	(00000)	(00000)	(0.0084)	(0.0003)	(00000)
Wealth above median	-0.4029	0.1007	1.5373	0.0971	0.5014	1.0084	0.5121	0.2010	0.1977
		(0.0046)	(00000)	(0.0046)	(0.0005)	(0.0011)	(0.0035)	(0.0011)	(0.0005)
Education < 4 yrs	-0.2988	1.3622	0.3203	0.1117	0.1782	1.3626	-0.2981	1.4606	0.1913
		(00000)	(00000)	(0.0005)	(0.0005)	(00000)	(0.0107)	(00000)	(00000)
Education = 4 yrs	-0.3963	1.2907	0.3142	0.1008	0.1933	1.1596	0.0016	1.0998	0.1944
		(0.0014)	(0.0079)	(0.0029)	(0.0018)	(00000)	(0.0019)	(00000)	(00000)
Education > 4 yrs	-0.5010	0.0993	0.8157	0.0994	0.2019	0.5006	0.4885	1.5321	0.2026
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0133)	(0.0005)	(00000)

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Stratification	logL				Parameter Esti	imates			
		γ_1	γ_2	Y	α	в	ш	ч	M
Whole sample	-0.3920	0.0906	0.7543	0.0940	0.4893	1.0306	0.0050	0.2331	0.2172
		(00000)	(00000)	(0.0003)	(0.0003)	(0.0003)	(0.0602)	(0.0003)	(00000)
Central	-0.4810	0.1012	0.8069	0660.0	0.5012	0.9983	0.5186	0.8082	0.4903
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1009)	(00002)	(00000)
Northeast	-0.3050	0.1037	1.5366	0.4134	0.1993	0.5021	-0.4453	0.2018	0.2018
		(0.0003)	(00000)	(00000)	(00001)	(00000)	(0.0637)	(00000)	(00000)
Wealth below median	-0.3791	0.0137	1.1901	0.1926	0.1526	0.3977	0.0079	0.0000	0.1706
		(00000)	(0.0865)	(00000)	(0.4319)	(0.1029)	(0.0889)	(00000)	(0.4319)
Wealth above median	-0.4110	0.1258	0.7342	0.1250	0.4204	0.9768	-0.0017	0.8564	0.5975
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0983)	(0.0003)	(00000)
Education < 4 yrs	-0.2987	0.1177	0.9560	0.1175	0.4453	0.9884	-0.2887	0.7864	0.5389
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1231)	(0.0004)	(00000)
Education = 4 yrs	-0.3887	0.0885	0.6860	0.1032	0.4754	0.9534	0.0037	0.4259	0.2770
		(00000)	(00000)	(0.0007)	(00000)	(00000)	(0.0723)	(00000)	(00000)
Education > 4 yrs	-0.5089	0.5855	1.4296	0.3867	0.1937	2.8396	-0.5814	0.2231	0.9863
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1015)	(00000)	(00000)

Table B.6: Borrowing and Lending, No Wealth Lotteries

Stratification	logL				Parameter Est	imates			
		γ_1	γ_2	У	α	в	Ш	ч	M
Whole sample	-0.3910	0.0969	0.4930	0.1071	0.5458	1.0113	0.0012	0.8222	0.4355
		(1.9838)	(1.9838)	(0.0002)	(1.0923)	(2.2650)	(0.0635)	(2.2905)	(1.9838)
Central	-0.4734	0.0910	0.9378	0.1049	0.4628	0.9959	0.4262	0.8381	0.5548
		(00000)	(00000)	(0.0072)	(0.0072)	(0.0108)	(0.1027)	(0.0108)	(0.0072)
Northeast	-0.3047	0.0973	1.5475	0.3990	0.1994	0.4999	-0.4451	0.2077	0.2041
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0641)	(00000)	(00000)
Wealth below median	-0.3793	0.6569	0.2990	0.1094	0.8125	3.7422	0.0005	0.5697	1.0869
		(0.0136)	(0.0136)	(0.0136)	(0.0136)	(00000)	(0.1246)	(00000)	(0.0125)
Wealth above median	-0.4028	0.1007	0.7937	0.4070	0.7958	2.9862	0.4981	0.2038	0.9964
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1125)	(00000)	(00000)
Education < 4 yrs	-0.2987	0.1060	0.7521	0.0902	0.4884	0.9331	-0.2887	1.0488	0.6010
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.1231)	(0.0003)	(00000)
Education $= 4$ yrs	-0.3896	0.1007	0.8042	0.0989	0.4990	1.0001	0.0001	0.8304	0.4957
		(0.0244)	(00000)	(0.0244)	(0.0587)	(0.0420)	(0.0775)	(0.0420)	(0.0244)
Education > 4 yrs	-0.5030	0.0970	1.6438	0.3933	0.5167	0.5063	0.3507	1.5491	0.2067
		(0.0012)	(0.0001)	(00000)	(0.0012)	(0.0037)	(0.1772)	(00000)	(0.0012)

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Stratification	logL				Parameter Esti	imates			
		γ_1	γ_2	У	α	б	ш	к	M
Whole comele	0.202.0		1 4660	0 1010		1 0002	(3) S (1)	1 4020	
WILDIE SALIPIE	0066.0-	0.0707	1.4000	0.124.0	0.7974	C000.1	-0.7272	4044.I	0.4712
		(00000)	(00000)	(00000)	(00000)	(00000)	(0.0317)	(00000)	(00000)
Central	-0.4808	0.0966	1.4102	0.5715	0.8767	2.8253	-0.6443	0.1990	0.4406
		(00000)	(00000)	(00000)	(0.0009)	(00000)	(0.0417)	(00000)	(00000)
Northeast	-0.3045	0.6042	0.3027	0.1003	0.2067	0.9905	-0.4741	0.7905	1.0238
		(0.0006)	(0.0036)	(00000)	(00000)	(00000)	(0.4683)	(00000)	(0.0001)
Wealth below median	-0.3789	1.2987	0.8019	0.1013	0.5012	0.9988	-0.5137	0.2005	0.2005
		(0.0029)	(0.0011)	(0.0011)	(3.7497)	(0.0007)	(0.9355)	(3.7510)	(00000)
Wealth above median	-0.4142	0.5830	0.3025	0.1011	0.2049	0.9861	0.5058	0.8093	0.5061
		(0.0001)	(00000)	(0.0004)	(00000)	(0.0011)	(0.4342)	(00001)	(00000)
Education < 4 yrs	-0.2985	0.5882	0.3016	0.1015	0.2029	1.0063	-0.5070	0.7884	0.5072
		(0.0008)	(0.0026)	(0.0428)	(0.0008)	(0.0214)	(0.3828)	(0.0216)	(00000)
Education = 4 yrs	-0.3915	0.1000	1.5124	0.4044	0.8090	1.0010	-0.5009	1.4967	0.5102
		(0.0001)	(0.0001)	(00001)	(00001)	(0.0001)	(0.0400)	(00001)	(00000)
Education > 4 yrs	-0.5092	0.0999	1.4735	0.5952	0.8091	2.9998	-0.5503	0.1977	0.4822
		(00000)	(00000)	(00001)	(00000)	(00000)	(0.0830)	(00000)	(00000)

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Stratification	logL				Parameter Est	imates			
		γ_1	γ_2	У	α	d	ш	к	M
Whole sample	-0.3924	0.1012	1.5167	0.3934	0.8088	1.0017	-0.5070	1.4970	0.5034
		(0.1441)	(0.0636)	(0.0035)	(0.0964)	(0.0072)	(0.0358)	(0.0048)	(0.1461)
Central	-0.4776	0.6020	0.8377	0.1002	0.7992	0.9975	-0.5014	1.4958	0.2000
		(0.0020)	(0.0079)	(0.0162)	(0.0011)	(0.0020)	(0.4080)	(0.0335)	(0.0037)
Northeast	-0.3037	0.5935	0.3028	0.1008	0.2024	1.0022	-0.5055	0.7848	1.0297
		(0.0114)	(0.0599)	(0.1767)	(0.1678)	(00000)	(0.5011)	(00000)	(00000)
Wealth below median	-0.3785	0.5985	1.5022	0.5993	0.5007	0.4999	-0.5007	0.7998	0.2097
		(0.0005)	(0.0003)	(0.0003)	(0.0022)	(0.0002)	(0.1602)	(0.0005)	(00000)
Wealth above median	-0.4054	0.1008	1.5114	0.4003	0.8061	0.9989	-0.5039	1.5099	0.5027
		(0.5075)	(0.0866)	(0.1319)	(0.1309)	(0.1309)	(0.0644)	(0.0867)	(0.5073)
Education < 4 yrs	-0.2988	0.6012	0.3013	0.1005	0.2004	0.9977	-0.5027	0.8185	1.0053
		(0.0181)	(0.0004)	(00000)	(0.0004)	(0.0001)	(0.7669)	(0610.0)	(00000)
Education $= 4$ yrs	-0.3907	0.1007	1.5137	0.4021	0.8075	1.0014	-0.5038	1.4838	0.5049
		(0.0465)	(0.0307)	(0.5478)	(0.0465)	(0.2987)	(0.0448)	(0.2016)	(00000)
Education > 4 yrs	-0.5054	1.3034	1.4814	0.6042	0.7861	1.0155	-0.5161	1.5127	0.2014
		(00000)	(0.0268)	(0.0104)	(0.0202)	(0.0329)	(0.1020)	(0.0134)	(0.0203)

Table B.9: Insurance, With Wealth Lotteries

	Vith Lotteries		946 ** 0.9102	806 1.1583	1217 ** -0.2764	1423 -1.0632	.400 0.5503	0.0920	535* 0.8791	.651 0.2171	766 1.7073 *	0.1033	939 0.9408	774 -0.4402	935 -0.5956	003 -0.1715
statistics	Contracts W	OL V BL OL	-2.5688 ** -1.9	-1.7124 * -0.7	-0.6869 -2.0	0.0357 -0.9	-0.2858 0.5	-0.1069 -0.0	-2.1029** -1.7	0.3138 0.4	-0.6726 0.3	-0.1079 0.0	-1.8734 * -1.2	-0.3331 -1.2	-0.2440 -1.2	0.5397 0.6
omparison Z-	otteries DNI INI	BINL V INL	0.9661	-0.6282	-0.1041	-0.1849	0.3952	-0.1611	0.9752	0.0463	0.4407	-0.1882	0.3873	0.9157	-0.2598	-0.6522
C	ts Without Lo	DINE V INE	-3.8197 **	-1.6860*	-1.2779	-0.1906	0.3785	-0.5392	-4.2849 **	0.3121	-3.2705 **	-0.7354	-3.2053 **	-0.2876	-0.5817	-2.4783 **
	Contract	SINE V BINE	-4.1471 **	-1.5358	-1.1582	-0.1354	-0.5959	-0.6780	-4.1975 **	0.1908	-3.9320**	-0.5545	-3.1712 **	-0.6656	-1.1406	-2.8807 **
Stratification			Whole sample	Central	Northeast	Wealth below median	Wealth above median	Education < 4 yrs	Education $= 4$ yrs	Education > 4 yrs	No formal credit	Formal credit	Any debt	No debt	Northeast, BAAC	Central, BAAC
% business			13.8%	19.2%	9.1%	12.6%	15.1%	9.0%	13.8%	21.1%	12.8%	19.2%	16.1%	10.4%	15.1%	26.0%
Z			2313	1091	1222	1157	1156	455	1554	304	1927	386	1388	925	231	150

Table B.10: Model Comparisons – Vuong Statistics

Z	% business	Stratification		Comparison	Z -statistics	
			Lotte	ry vs. No Lotte	ery	
			SNL v SL	BNL v BL	INL v IL	BL v INL
2313	13.8%	Whole sample	-3.4909**	-0.7388	-1.5230	1.4123
1091	19.2%	Central	-1.8316*	-1.8850*	-0.7869	1.9989 **
1222	9.1%	Northeast	-0.9802	-0.3858	-0.7900	-0.0385
1157	12.6%	Wealth below median	0.0602	0.4592	-0.3609	-0.3314
1156	15.1%	Wealth above median	-1.2570	-1.2758	-2.4186**	1.6954 *
455	9.0%	Education < 4 yrs	-0.7483	0.0000	0.1864	-0.0424
1554	13.8%	Education $= 4$ yrs	-3.3584 **	0.4844	-0.5525	-1.7535*
304	21.1%	Education > 4 yrs	-0.4867	-0.4291	-0.7295	0.4651
1927	12.8%	No formal credit	-3.6252 **	-1.2229	-0.1956	0.3766
386	19.2%	Formal credit	-0.9870	-0.7525	-0.0329	0.0152
1388	16.1%	Any debt	-3.3626 **	-1.1320	-1.1727	-1.2939
925	10.4%	No debt	-0.6104	-0.1521	-1.6061	-1.2774
231	15.1%	Northeast, BAAC	0.1186	0.5674	0.0893	-1.2935
150	26.0%	Central, BAAC	-2.5230**	-1.0820	-0.2479	0.6003

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Table B.10 (cont.): Model Comparisons – Vuong Statistics

	Data			Model F	legimes		
		Sav. No Lot.	Sav. With Lot.	Borr. No Lot.	Borr. With Lot.	Ins. No Lot.	Ins. With Lot.
Q1	62	80	69	59	69	72	75
Q2	67	109	69	69	69	73	75
03	56	160	69	69	69	73	75
Q4	118	267	62	117	109	102	107
Total	320	616	286	314	316	320	332
Log-L	0	-0.4121	-0.3966	-0.3920	-0.3910	-0.3950	-0.3924

Table B.11: Actual and Predicted Number of Entrepreneurs By Wealth Quartile (Whole Sample)

Observations	1,325
Mean wealth (Baht)	65,123
Median wealth	42,971
Wealth standard deviation	80,770
Wealth skewness	5.2800
Maximum wealth	1,322,433
Minimum wealth	2,599
Fraction with business	17%
Mean wealth (with business)	82,620
Mean wealth (no business)	61,467
% in business, wealth >= median	24%
% in business, wealth < median	11%

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Model	logL	γ_1	γ_2	r	a	d	m	К	w
SL	-0.3647	0.1016	0.8272	0.0972	0.1904	0.9493	-0.5747	0.2033	0.4929
		(00000)	(0.0012)	(00000)	(0.0008)	(0.0003)	(4.2260)	(00000)	(00000)
BL	-0.3654	0.1005	1.5047	0.4017	0.1982	1.0007	-0.5032	0.2003	0.5032
		(00000)	(00000)	(00000)	(00000)	(00000)	(3.9938)	(00000)	(00000)
IL	-0.3682	0.1397	1.4711	0.2137	0.7352	0.9790	-0.0076	1.7697	0.6669
		(0.1464)	(00000)	(0.1464)	(0.1464)	(0.1464)	(3.6068)	(0.0828)	(00000)

Table B.13: Static Parameter Estimates, 1976 SES Young Households Data, Whole Sample

Stratification	BL v SL	SL v IL	BL v IL
Whole sample	-0.2297*	0.7722	0.5865
Wealth below median	-0.4875	0.3914	0.1179
Wealth above median	-1.1841	0.3729	-0.7878

*The reported values are Z-statistics.

Table B.14: Model Comparisons, 1976 SES Young Household Data

Model	logL	S	1+g	Metric
SL	-0.3647	0.3221	1.0000	1.6921
BL	-0.3654	0.4672	1.0000	1.4672
IL	-0.3682	0.2928	1.0000	1.3296

Table B.15: Dynamic Parameter Estimates, 1976 SES Young Household Data